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A STRATEGIC APPROACH TO FINANCIAL OPTIONS

Abstract

To explain the strategic dimension of option pricing, it would be useful to address the very concept of the option. Options allow postponing the decision to a future point of time. This flexibility gives an opportunity to account for additional information. The flexibility of decision-making has its advantages and benefits as well. The article analyses the problem of option pricing on the example of probable states of nature. The authors explain how these possible states change the strategic choice of the player.

Key words:

Financial option, game, option premium, real option.

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Introduction

We claim that financial market risks are for the most part behavioural risks and not risks of the state of nature. Financial management -- based on natural sciences -- assumes condition about risks is faulty when behavioural risks are intentionally or unintentionally overlooked. In the area of financial engineering, there are several spectacular blunders of risk management. We will only address two cases here. One example is the rise and fall of the hedge fund *Long Term Capital Management* (LTCM). Within a few months up until August 1998, LTCM generated losses amounting to 4 billion US-Dollars. In Germany, people remember the *Metallgesellschaft AG* in autumn 1993. At that time, the subsidiary *Metallgesellschaft Refining and Marketing Inc.* (MGRM) went bankrupt and generated a total loss of 1.5 billion US-Dollars for the parent company. The reasons for both cases are identical. The finance managers lost touch with the realities of the financial markets. Nowadays, the predominant practice of decision-making in the financial sector is inevitably leading to comparable situations.

We thus question whether it is sensible to place trust in a theory which systematically ignores the strategic behaviour of the markets. Game theory shows that behavioural risks can indeed be calculated. From a game theory perspective, it does not suffice to uncritically use a time series of historical market data as input for the risk calculation. The value at risk model, the approach recommended by the banking supervision, cannot be applied as a universal model. Only in exceptional cases, such as insurance products, do non-strategic risk models offer optimal solutions. In our view, however, the financial sector is clearly dominated by behavioural risks. Behavioural risks are strategic risks. With his game theory, the Nobel Prize holder John Nash developed the universal solution concept for strategic games. However, the translation of the game theory concepts into concrete guidelines for action has yet to be explained. The research departments of banks should concentrate their activities on such questions. Game theory enables us to illustrate and mathematically solve interdependent decision-making situations, which are known as strategic games. In game theory, every player puts him or herself in the shoes of the other player. He or she analyzes the potential consequences of his/her decision and uses this knowledge to make the best decision possible. Below, we would like to provide a simple example of the mentioned interrelations.

Howard's Party Problem

To explain the strategic dimension in pricing options, it will be helpful to go back to the heart of the idea behind the concept of an option: First of all, options open up the possibility to postpone current decisions to a future point of time. Because of this flexibility, additional information and new experiences can be taken into consideration. Of course, there are advantages and benefits resulting from this flexibility. The value of the flexibility is reflected in the positive option price. The discussion on real options is emphasizing this feature very much.

To give this aspect more intuition, we have a brief look at the Party Problem in Howard (1996). This problem is a pure problem of choice under uncertainty, that is to say, a pure decision problem in the absence of any strategic interaction. The next chapter will put the party problem into a strategic perspective.

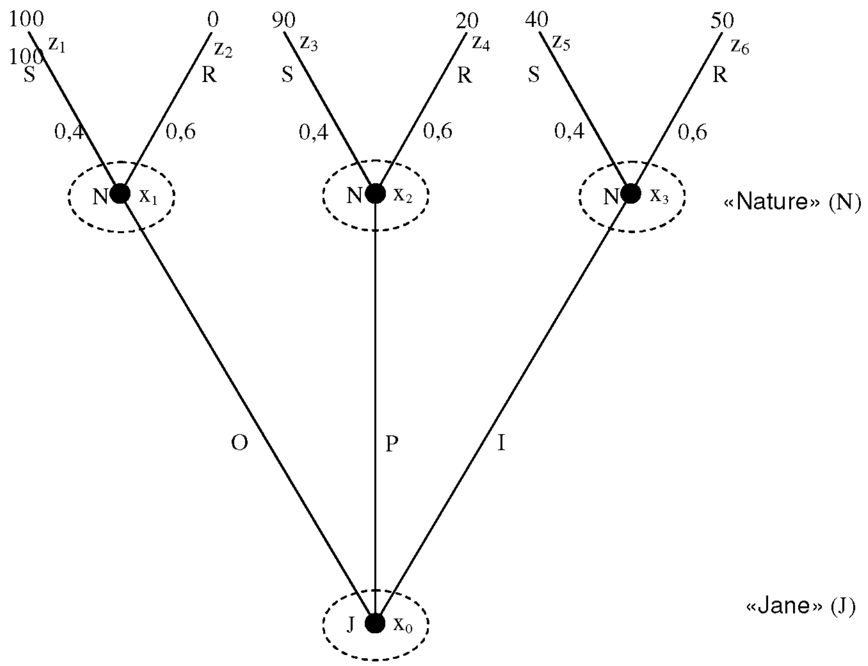
At time $t = 0$, Jane is planning a party that is to come off at time $t = 1$. She has three alternatives: having the party outdoors (O), on the porch (P), or indoors (I). The decision on the location must be taken at $t = 0$. At $t = 1$, nature will choose one of two possible states: weather it will be sunny (S) or rainy (R). Let us assume that Jane assigns probability .4 to S and .6 to R. The money values to the various combinations of location and weather are shown in Figure 1. In addition, Jane is risk neutral by assumption.

In this figure, x_i denotes a decision node, $i = 0, 1, 2, 3$. At x_i, $i = 1, 2, 3$, nature makes a chance move: alternative S with probability .4 and alternative R with probability .6. At x_0, Jane's decision node, Jane chooses one of her alternatives O, P, or I. After nature's move, the «game against nature» is completely represented by the terminal nodes z_i, $i = 1,...,6$. At each terminal node of this one-person decision problem, a monetary payoff to Jane is listed arising from the sequence of moves leading to that terminal node.

By choosing one of her alternatives O, P, and I at x_0, Jane actually chooses one of three possible lotteries: L_O, L_P, and L_I, being defined by $L_O := (100, 0; .4, .6)$, $L_P := (90, 20; .4, .6)$, and $L_I := (40, 50; .4, .6)$. Let the corresponding expected return of each lottery be denoted by E_O, E_P, and E_I respectively. Since we have $E_O = 40$, $E_P = 48$, and $E_I = 46$, Jane, being risk-neutral, will choose the alternative P at her decision node x_0.

Figure 1

Jane's Decision Tree



The Party Problem and the Option Idea

In the preceding section, the decision on the party's location has to be taken today, at time $t = 0$. Now suppose Jane has been offered the possibility to decide on the party's place after the state of nature has been disclosed, that is at $t = 1$. This comes to offering her an option: The offer permits a future decision following the revelation of information. From an alternative perspective this means that Jane has access to an early warning system. The question is: What is the maximum price Jane is willing to pay in order to get access to the warning system?

Holding this option for Jane means switching to a new lottery $L_* = (100, 50; .4, .6)$. The reason is simple. Knowing the state of nature, if it is sunny (*S*), Jane will choose the alternative *O*, the outdoor party, and, if it is rainy (*R*), she will choose *I*, the indoor party, which gives her payoffs of 100 and 50 respectively. In the following, this new lottery is referred to as the «option lottery».

If Jane has access to this option, she has a new lottery available with an expected return of 70. Without this option she could realize an expected return of 48 at best. Hence, the value of this option is 22 (= 70 – 48). This difference is the maximum price Jane is willing to pay to get access to the new lottery. Note that the value of the option varies with the probabilities of the two states of nature occurring. These probabilities will turn out the strategic decision variables of a new player as explained in the next section.

Strategic Option Pricing: The Theoretical Background

Now think of the hypothetical situation in which the «rainmaker» St. Peter enters the stage. St. Peter makes his choice within a set of probability distributions over the two possible states *S* and *R*. His decision variables are the two probabilities for the states of nature. St. Peter has a good incentive to do so because he can expect to get paid a substantial share of the benefit of 22 calculated above.

Consequently, the probabilities of the different states of nature are no longer exogenously given. Moreover, St. Peter is in a position to make the offer that permits Jane a decision on the party's locality at time $t = 1$ following the revelation of the true state of nature. Given this scenario, Howard's party problem becomes a non-cooperative two-person game, with St. Peter as player 1 and Jane as player 2.

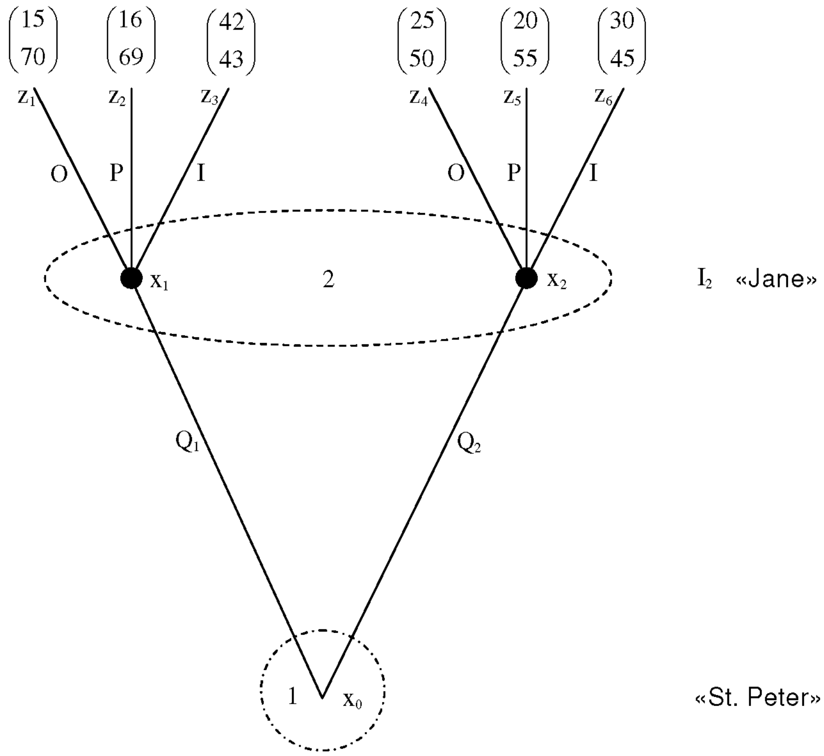
We are going to discuss a simple case in which player 1 has only two strategies: $Q_1 := (q_1, 1 - q_1)$ and $Q_2 := (q_2, 1 - q_2)$. The choice of strategy Q_i means that states S and R will be realized with probabilities q_i and $1 - q_i$ respectively, $i = 1, 2$.

As before, the set of strategies of player 2 consists of the alternatives *O*, *P*, and *I*. To be more intuitive about the introduction of the strategic feature into Howard's party problem, let us pick (without loss of generality) two alternatives: Q_1 :=(.7, .3) and Q_2 := (.5, .5). The strategic interaction between player 1 and player 2 is demonstrated by the following game representation in the extensive form (Figure 2).

The game starts at an initial decision node, x_0, where player 1 makes his move, deciding whether to let the sun shine with probability .7, or to let it shine with probability .5. The nodes x_1 and x_2 are player 2's decision nodes. We have drawn a circle around the nodes to indicate that these two nodes are elements in single «information set» (I_2). The meaning of this information set is that when it is player 2's turn to move, she cannot tell which of these two nodes she is at because she has not observed player 1's previous move.

Figure 2

A Strategic Extension of Howard's Party Problem



The terminal nodes z_i, i = 1, ..., 6, indicate the end of the game, and an outcome is assigned to each terminal node. We think of the players attaching monetary values to the various outcomes. At each terminal node, we list the players' monetary payoffs arising from the sequence of moves leading to that terminal node. The first component of the respective payoff vector refers to player 1's payoff and the second to player 2's payoff. We assume that player 1, St. Peter, is risk-neutral as well.

Player 2's payoffs at the terminal nodes are determined as follows: Suppose player 1 had chosen the alternative Q_1 at x_0. If player 2, who is uninformed about the choice of player 1, chooses the alternative O at her information set I_2, the lottery $L(O, Q_1)$ defined by $L(O, Q_1)$: = (100, 0; .7, .3) is put into effect (see Figure 1). The expected return of this lottery is 70. This gives player 2 a payoff of 70 at the terminal node z_1.

In a comparable way, the terminal nodes z_2 and z_3 are connected with the lotteries $L(P, Q_1)$ and $L(I, Q_1)$ respectively, where $L(P, Q_1)$ is defined by $L(P, Q_1)$:= (90, 20; .7, .3) and $L(I, Q_1)$ by $L(I, Q_1)$:= (40, 50; .7, .3) (see Figure 1). The expected returns of these lotteries are 69 and 43 respectively, which gives a payoff to player 2 of 69 at the terminal node z_2 and of 43 at z_3.

The payoffs to player 2 at the terminal nodes z_j, j = 4, 5, 6, result in similar way from the expected returns of the lotteries $L(O, Q_2)$, $L(P, Q_2)$, and $L(I, Q_2)$ respectively, being defined by $L(O, Q_2)$:= (100, 0; .5, .5), $L(P, Q_2)$:= (90, 20; .5, .5), and $L(I, Q_2)$:= (40, 50; .5, .5) (see Figure 1). Hence player 2's payoffs at the terminal nodes z_4, z_5, and z_6 are 50, 55, and 45 respectively.

Player 1's payoffs at the terminal nodes are calculated in the following way: Imagine that player 1, St. Peter, offers player 2, Jane, the possibility to decide on the party's location after the true state of nature will be disclosed. If player 2 accepted this offer, player 1 should then be given the corresponding value of the option as his payoff.

If, for instance, player 1 were committed to the alternative Q_1 = (.7, .3) at x_0, the corresponding «option lottery» $L_*(Q_1)$ is given by $L_*(Q_1)$:=(100, 50; .7, .3): This lottery describes the situation into which player 2 is put, if it were possible for her to decide on the party's locality after the true state of nature will be revealed and the probabilities of the two possible states are given by Q_1.

The expected return of this lottery is 85. The value of this option for player 2 at the terminal node z_1, that is in case she had chosen the alternative O at time $t = 0$ (choice of O at her information set I_2) is given by the difference 15 (= 85 – 70). Therefore player 1's payoff at z_1 is fixed at 15. Player 1's payoffs at z_2 and z_3 are fixed in a parallel way at 16 and 42 respectively: note that the value of the option in question for player 2 at the terminal node z_2 is given by the difference 16 (= 85 – 69) and at z_3 by the difference 42 (= 85 – 43).

If player 1, instead, were committed to his alternative Q_2 = (.5, .5) at x_0, the corresponding option lottery $L_*(Q_2)$ takes on the form $L_*(Q_2)$:= (100, 50; .5, .5) with an expected return of 75. Thus, the hypothetical option price at the terminal nodes z_4, z_5, and z_6 is given by 25 (=75 – 50), 20 (= 75 – 55), and 30 (= 75 – 45) respectively. The (uniquely determined pure strategy) Nash equilibrium of this hypothetical game will serve as a benchmark for strategically determining an option premium for the extended party problem of Howard (1996). (Note: Nash equilibrium occurs when each player makes his or her best response. A player's best response is the strategy that maximizes that player's payoff, given the strategies of other players)

Let q_* denote the probability with which St. Peter will realize in equilibrium state S and let J_* denote the equilibrium payoff to Jane. Finally, let E_* be the expected return of the option lottery determined by the equilibrium, i. e., E_* is the expected return of the lottery L_* given by L_* = (100, 50; q_*, $1 - q_*$). Then, the strategically determined price for the option that permits a decision on the party's locality after the state of nature will be disclosed is given by $E_* - J_*$. Note that

$E_* - J_*$ is just the equilibrium payoff to St. Peter in our hypothetical game discussed above.

We are in a position to derive the following properties:

1) There is a uniquely determined non-cooperative equilibrium in the preceding «option-premium game» which is given by the pair (Q_2, *P*). First observe that strategy *I* of player 2 is dominated by her strategy *P* and thus can be eliminated. The game of Figure 2 can therefore be reduced to the following strategically equivalent 2 × 2 – game.

Figure 3

The Reduced Form of the «Option Premium Game»

1 \ 2	O	P
Q_1	15 ; 70	16 ; 69
Q_2	25 ; 50	20 ; 55

It is easy to see that Q_2 is a best reply of player 1 to the strategy choice *P* of player 2, and vice versa. This is a unique Nash equilibrium. As a result, we obtain a strategically determined premium for the option of 20 (= 75 – 55). The details of the lottery and the payoffs are given by: $L_* = (100, 50; .5, .5)$, $E_* = 75$, $J_* = 55$.

2) Note that at this option price player 2 is just indifferent between the equilibrium of the preceding game giving her the (expected) payoff $J_* = 55$ and the situation of the «equilibrium option lottery», giving her the (expected) payoff of 75, but only at a price of 20 (the maximum price player 2 is willing to pay in equilibrium to be in the option lottery situation).

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