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MORAL HAZARD CONTRACTING AND CREDIT RATIONING IN OPAQUE CREDIT MARKETS

Abstract

We make a first step in the literature to analyze a hybrid model of credit rationing with simultaneous presence of adverse selection and moral hazard. Motivated by the observation that credit markets in less developed countries are rather opaque due to the lack of necessary institutions to facilitate information sharing among lenders, we re-examine the issue of credit rationing in such an environment. For a range of different parameter values we fully characterize the sub game perfect equilibria of the loan contracting game. Under certain parameter values there is type II credit rationing for some borrowers and credit forcing for others. Credit forcing is shown to be efficient in a constrained sense. The results are contrasted with those in DeMeza and Webb (1992).

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Key words:

Credit Rationing, Moral Hazard, Adverse Selection.

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1. Introduction

A large body of literature has developed that seeks to explain the phenomenon of credit rationing, by appealing to the existence of asymmetric information in the credit market. There are basically two kinds of explanations in this line of research: explanations based on adverse selection and those based on moral hazard. The seminal work of Stiglitz and Weiss (1981) is an example of the former. In their model, the bank cannot observe the riskiness of a firm's project, and the return to the bank is affected by the possibility of firm's default. They argue that, under limited liability on the part of the firm, raising the interest rate of the loan does not necessarily increase the return to the bank because, when facing a higher interest rate, only those firms with more risky projects will still demand credit while the less risky ones will drop out of the market. This adverse selection effect prevents the bank from raising interest rate to eliminate the excess demand in the market and credit rationing ensues.

As an example of the moral hazard approach, Bester and Hellwig (1987) consider the possibility of credit rationing as a result of borrowers' hidden action. The borrower can choose between a «good» and «bad» investment project, characterized by the projects' different return distributions (i.e. riskiness), after having obtained the funds from the lender. With limited liability the borrower's project choice has an impact on the return to the lender, yet the loan contract cannot prescribe the choice before the project is undertaken. Under such circumstances credit rationing again arises, this time as a consequence of the post-contractual+++ informational asymmetry between the lender and the borrower, as opposed to the hidden information in the above model of Stiglitz and Weiss which represents the pre-contractual informational asymmetry.

Though differing in their special form of hidden information or hidden action, other models in the literature are all based on either adverse selection or moral hazard in their explanation of credit rationing, that is, the incentive structures in those models are all one-dimensional. In many real-world applications, however, pure one-dimensional adverse selection or pure one-dimensional moral hazard can hardly capture the essence of the economic problem involved. There is clearly a need to study multi-dimensional incentive structures in their relation to credit rationing. Hellmann and Stiglitz (2000) attempt a first step in this direction by introducing two-dimensional private information (about both the expected return and risk of the firm's project) in a model of credit and equity rationing. As it turns out, the introduction of the richer form of informational asymmetry leads to some conclusions that are different from those in previous work on the subject. For example, they show that credit and equity rationing are compatible in their environment, in contrast with the results of DeMeza and Webb (1987) where either credit or equity rationing disappears when the asymmetric information is one-dimensional about either the expected return or risk of the project.

In this paper we introduce multi-dimensional incentive structure in a different direction in our study of credit rationing. We allow adverse selection and moral hazard to be simultaneously present in a hybrid model. The particular form of pre-contractual informational asymmetry is motivated by the observation that credit markets in many less developed countries are rather opaque in the sense that a lender may not easily observe a borrower's financial status when the latter applies for a loan. This hidden information of the borrower, in the form of pre-existing debts, poses a risk on the lender because it affects the borrower's ability to repay the debt. We model the loan contracting problem between the lender and the borrower as a screening game in which the borrower not only self-selects the lender's contract offers but also subsequently chooses a level of work effort optimally. Since the return to the lender depends partially on the borrower's choice of work effort in a stochastic fashion, this generates endogenously the risk faced by the lender. This endogeneity of risk in our model thus stands in contrast with the exogenous risk in other credit rationing models in the literature. For example, in the above papers of Stiglitz and Weiss (1981) and Bester and Hellwig (1987), the riskiness of firm's project is simply assumed as an exogenous element of their respective model. The risks in the models of Milde and Riley (1988) and Gale and Hellwig (1985) are also exogenous; the former introduces risk with an abstract random variable while the latter interprets it as arising from, say, the uncertainty about the future price of entrepreneur's output.

In addition to the issue of exogenous vs. endogenous risk in credit rationing models, a criticism has been raised about the exogenous nature of the loan contract between the lender and the borrower. In early research on credit rationing the loan contract is exogenously specified as a standard debt contract which requires the borrower to repay a fixed pre-specified amount, and is offered to all potential borrowers without discrimination. To address this problem Wette (1983), Bester (1985a) and Besanko and Thakor (1987), for example, explore the idea of screening by collateral requirements in which borrowers self-select the lender's loan offers consisting of the interest rate and an associated collateral requirement. Bester (1985b), Milde and Riley (1988), and Grinblatt and Hwang (1989), on the other hand, introduce variable loan sizes as a screening device for the lender. In our model, loan size is also variable which the lender uses as a second instrument besides the interest rate to screen borrowers.

The literature, following Keeton (1979), distinguishes between two different types of credit rationing. Type I credit rationing is said to occur when borrowers obtain a smaller loan than they would like to get at the lender's quoted interest rate. Type II credit rationing, on the other hand, refers to a situation where, among a population of observationally indistinguishable borrowers, some borrowers obtain a loan from the lender while others do not. With fixed investment needs by all potential borrowers it is not possible to address the issue of type I credit rationing because all the lender's loan offers will be of an all-or-nothing nature. In our environment, under certain parameter values of the model, we show that type II credit rationing exists for one type of borrowers, while for the other type of borrowers a phenomenon opposite to type I credit rationing, namely, credit forcing occurs in the credit market: the borrower is «forced» to accept a larger loan than she wishes to obtain at the lender's quoted interest rate. While the loan size granted to the borrower is not the most desirable from her point of view, it is nevertheless the constrained optimum level of investment. In these respects our paper is close to De-Meza and Webb (1992) although they study an environment with symmetric information and exogenous risk and their focus is exclusively on type I credit rationing. The results we obtain are in contrast with theirs, on issues such as whether or not credit rationing can arise in a monopolistic market as well as the role of unobservability of borrowers' indebtedness and the priority rule for debt repayment in credit rationing. One distinctive feature of our analysis is that we unambiguously establish the existence (or non-existence) of credit rationing for a range of different parameter values in the model, whereas most previous papers in this area merely establish the *possibility* of credit rationing. Indeed, as Hellmann and Stiglitz (2000) put it, «ideally one would like to have a general characterization of how these parameters translate into the existence of rationing equilibria» but «this turns out to be analytically not tractable [in our model].» From a methodological point of view, then, our paper contributes to the existing literature by exploring a fairly general yet tractable model of credit rationing.

This paper is also related to the literature on non-exclusive contracts. Park (2004), for example, considers the moral hazard contracting problem between one single borrower and a lender in a model in which the borrower decides on an interim wealth level which subsequently becomes her private information through borrowing in an outside credit market, before contracting with the new lender. In this respect our model is similar to his in that the borrower's pre-existing debt is also her private information. Bizer and DeMarzo (1992), in their study of sequential banking, considers an environment in which the lender cannot observe the borrower's future borrowing from other lenders; in our environment, by comparison, it is the borrower's past borrowing from prior lenders that is unobservable to the current lender.

The rest of the paper is organized as follows. Section 2 describes the nature and cause of opaque credit markets and presents the formal model. Section 3 derives the optimal contracts under symmetric information as a benchmark case. We analyze in Section 4 the screening equilibria with unobservable debts. Section 5 provides an examination of the implications of the screening equilibria for credit rationing. Concluding remarks are contained in Section 6.

2. The Environment

2.1. Opaque Credit Markets and the Coexistence of Multiple Debts

Borrowers in the credit markets often have multiple sources from which to obtain funds. The consumer credit market provides a good example. In the United States, it is common for consumers to hold several credit cards issued by different lending banks. Bisin and Guaitoli (2004) estimate that on average a typical American household has more than seven credit cards. Most credit card debts are unsecured and thus are subject to default. As documented by Petersen and Rajan (1994, 1995), small businesses are also frequently able to borrow from multiple lenders. In Europe, multiple sources of credit are even more prevalent (Detragiache, Garella and Guiso, 2000). In all these instances debt owed by a borrower to one lender imposes an externality on another lender, because the higher the borrower's indebtedness the higher is the risk of default to a lender.

While it is practically impossible to restrict borrowers' access to multiple sources for loans, institutions have been created to alleviate the problem of informational asymmetry between the lenders and borrowers which may arise in their absence. Credit bureaus are such an example. Lenders participating as members of a credit bureau share information, for example, on their common debtors' financial status. When a consumer applies for a line of credit from a bank, her total indebtedness to date, among other things, will be checked by the bank using information gathered from various credit bureaus. If it is determined that her total debt is too high relative to her perceived ability to repay (e. g., as indicated by her annual income), the application will likely be turned down.

Although countries like the United States and Britain have a relatively mature system of consumer information sharing among lenders, in other countries like Belgium, Italy and Spain, such information sharing is minimal (Pagano and

Jappelli, 1993). And in some less developed countries such information sharing is virtually non-existent which partly accounts for the slow development of the consumer credit industry in these countries[1]. Furthermore, in most countries, there are regulations credit bureaus must observe which forbid the collection of certain types of consumer information; for example, information on debts owed to friends, family members and other private money lenders is generally not collected by credit bureaus. Though in principle lenders themselves may try to find out about a borrower's private debts by incurring a cost, such cost may prove so prohibitively high as to render the practice impossible[2].

In light of these institutional realities in the credit markets we model a borrower's indebtedness at the time of applying for a new loan as unobservable to the lender, following Bizer and DeMarzo (1992)[3]. We also assume in the event of a borrower's bankruptcy that she pays off her prior debts with whatever she is able to, before repaying the new lender's loan. This assumption can be justified if debts are prioritized, i. e., if senior debts retain priority over junior ones in getting repaid in bankruptcy. Alternatively, even if debts are not prioritized in the legal sense, they are nevertheless prioritized from the borrower's perspective; that is, when it comes to repaying debts, the borrower's preferences are such that earlier creditors get repaid before later ones can receive any payment. This is particularly true in many less developed countries where a large fraction of consumer debts are those owed to private parties such as friends and family members of the borrower. In the absence of necessary legal institution to enforce debt repayment in those countries, the borrower has both the incentive and the means to make sure that debts owed to her «preferred» lenders get paid off first[4].

2.2. The Loan Contract

The credit market consists of a monopolist lender and a large population of borrowers. The borrowers for some reason have incurred a certain amount of debt in the past, and are currently in a state of financial distress. They do, however, own a productive technology which, if funded by appropriate amounts of investment, can produce sufficient output to pay off the existing debts. Borrowers

[1] According to Iwasaki (2004), in China, among all bank cards issued as a mean of payment, only 5% are credit cards in the usual sense; the rest are what one will call debit cards in the US which require the card holder to maintain deposits with the issuing bank. See, also, Li et al (2005).

[2] The related literature on costly state verification addresses a similar point; see, for example, Townsend (1979), Gale and Hellwig (1985), and Williamson (1986, 1987).

[3] In the context of sovereign debt Kletzer (1984) studies a model in which banks are unable to observe borrowers' total indebtedness.

[4] See Longhofer and Carlstrom (1995) and Longhofer (1997) for a related issue.

differ in their indebtedness: some have a pre-existing debt d_L, while others have a pre-existing debt d_H, with $0 < d_L < d_H$. Borrowers are otherwise identical as described below, and they are henceforth referred to as *L*-borrowers and *H*-borrowers respectively according to the level of their pre-existing debt.

With a first-period investment $I \in [0,\infty)$, each borrower using her technology and an input of work effort $e \in [0,1)$ can produce a second-period output $F(I,e)$. Output increases with effort level in the sense of first-order stochastic dominance. Specifically, we assume that F has a multiplicative form: $F(I,e) = f(I)Z(e)$, where f satisfies $f(0) = 0$, $f'(0) = \infty$, $f' > 0$, $f'(\infty) = 0$ and $f'' < 0$, and is a 0–1 random variable with $prob\{Z(e) = 1\} = e$, and $prob\{Z(e) = 0\} = 1 - e$, i. e., a higher effort level leads to higher probability of success for any fixed amount of investment. Each borrower has the following utility function: $U(w,e) = u(w) - g(e)$, defined over her second-period wealth w and effort level e, where u satisfies $u(0) = 0, u' > 0$ and $u'' < 0$, and g satisfies $g(0) = 0$, $g(1) = \infty$, $g'(0) = 0$, $g'' > 0$ and $g''' > 0$[5]. Thus the borrower is risk-averse in wealth and a higher effort level costs her more in utility terms.

The lender is risk-neutral who operates in the market attempting to maximize his expected second-period profit. He raises loanable funds on the deposit market where the supply of funds is infinitely elastic at interest rate ρ. This means that the lender may obtain funds in any amount he desires at that rate should he decide to invest in a borrower's technology[6]. Without loss of generality we assume $\rho = 0$.

All parties have limited liability. First, the borrower assumes limited liability to the lender. That is, if the borrower goes bankrupt in the second period her final wealth is guaranteed no less than zero. Second, the lender has limited liability to the borrower's prior creditors: when the borrower fails to repay her prior debts the new lender cannot be held responsible to pay off those debts on behalf of the borrower. Finally, the lender's liability to his depositors is also limited.

We model this investment process as a two-stage game as follows. In the first period, after having identified an investment opportunity, the lender raises funds on the deposit market and subsequently offers a loan contract, (*I, r*), to the

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[5] An example of g satisfying these properties is $g(x) = -\ln(1 - x) - x$, $0 \le x < 1$.

[6] Early literature on credit rationing relies on the availability doctrine to explain the phenomenon: it is the limited availability of loanable funds that causes banks to ration credit. The assumption of an infinitely elastic supply of loanable funds makes arguments based on availability doctrine irrelevant and allows one to focus on the role of asymmetric information in causing credit rationing.

borrower, where l is the size of the loan and r the interest rate.[7] When offered a loan contract the borrower either declines the offer, in which case she ends up with zero final wealth and hence zero utility in the second period; or she accepts the offer, in which case she heads off to work and privately chooses an effort level e to produce a second-period output. If she does accept the offer then, in the second period, she repays her debts subject to limited liability and the priority rule for debt repayment (i.e., earlier debts get repaid before later ones can). For simplicity we adopt a tie-breaking rule by assuming that the borrower always declines a contract offer whenever she if indifferent between declining and accepting the offer, and the lender always chooses not to make an offer whenever he is indifferent between making and not making the offer.

3. Optimal Loan Contracts with Observable Debts

We wish to characterize the subgame perfect equilibrium (SPE) of this contracting game and in the process derive the lender's optimal loan contracts. Subgame perfection requires that the borrower's strategy cannot prescribe that she forgo the contracting opportunity with the lender when the latter's loan offer in the first period will give her a positive utility level in the second period. This requirement effectively eliminates empty threats by the borrower in order to get better loan terms in the equilibrium. In this section, we first examine as a benchmark case the situation in which the borrower's private information about her prior debt is observable to the lender.

When a loan contract is offered to the borrower, she must decide whether to accept or decline the offer. The following lemma establishes the condition under which the borrower will accept the lender's loan contract.

Lemma 1*. An i-borrower will accept the loan contract* (l_i, r_i) *if and only if* $f(l_i) - d_i - (1 + r_i)l_i > 0$ *. When this condition is satisfied the borrower chooses an effort level* e_i *which is the unique solution to* $g'(e_i) = u(f(l_i) - d_i - (1 + r_i)l_i)$ *.*

Proof. See Appendix.

Having established the necessary and sufficient condition for the borrower's acceptance of a loan contract, we now turn to the analysis of the SPE of the screening game under various conditions concerning the parameter values of the model. But before proceeding we first establish another lemma and define

[7] As pointed out by DeMeza and Webb (1992), in banking practice borrowers are usually quoted an interest rate and a loan size simultaneously. This is also true of credit card offers: applicants are provided with an interest rate together with a credit line.

two quantities, l and h, that will be needed frequently in later analyses. The lemma is straightforward so we state it without proof.

Lemma 2. *There exists a unique $l^* \in [0,\infty)$ such that $f(l^*) - l^* \geq f(l) - l$ for any $l \in [0,\infty)$. Moreover $l^* > 0, f'(l^*) = 1$ and $h^* \equiv f(l^*) - l^* > 0$.*

As the pre-existing debts are observable the lender can perfectly discriminate among the borrowers by offering each type of borrowers a possibly different loan contract. Since the lender is risk neutral maximizing his total expected profit is equivalent to maximizing his expected profit from each borrower type, whether the returns to investments in the two borrower types are independent or not. The lender's expected profit from the *i*-borrower, if the latter accepts the contract (l_i, r_i) and chooses an effort level e_i as given in Lemma 1, is given by

$$\pi_i = E[\max\{\min\{\max\{f(l_i)Z(e_i) - d_i, 0\}, (1+r_i)l_i\} - l_i, 0\} \mid e_i]. \tag{1}$$

Note that the two max's in the above expression, in their order of appearance, reflect respectively the limited liability of the lender to his depositors and that to the borrower's prior creditors. The *i*-borrower's expected second-period utility, if she accepts the loan contract (l_i, r_i) with $f(l_i) - d_i - (1+r_i)l_i > 0$ and chooses an effort level e_i, is given by

$$E_i = e_i \cdot u(f(l_i) - d_i - (1+r_i)l_i) - g(e_i) \tag{2}$$

We derive the SPE for each of the following three cases in turn. Case 1: $h^* \leq d_L$; Case 2: $d_L < h^* \leq d_H$; and Case3: $d_H < h^*$.

Case 1: $h^* \leq d_L$. Then $h^* \leq d_i, i = L, H$. Since, by Lemma 2, l is the unique maximum for $f(l) - l$ and $h^* = f(l^*) - l^*$, we have, for any contract offer (l_i, r_i), that $f(l_i) - d_i - (1+r_i)l_i \leq f(l^*) - l^* - d_i - r_i l_i = h^* - d_i - r_i l_i < 0$. By Lemma 1, neither borrower type will accept the contract, and consequently the lender will make offer to neither borrower type. This leads to the following theorem.

Theorem 1. *If $h \leq d_L$, then there is a unique SPE outcome of the game in which the lender makes no contract offer to either borrower type.*

Case 2: $d_L < h^* \leq d_H$. First of all, the *H*-borrower will decline any offer from the lender, and thus the lender will make no contract offer to this borrower type in the SPE, as the analysis in Case 1 shows. For the *L*-borrower, knowing that by offering a contract (l_L, r_L) with $f(l_L) - d_L - (1+r_L)l_L \leq 0$ he will earn a zero profit in the second period because the *L*-borrower will decline such an offer, the lender will try to offer (l_L, r_L) such that $f(l_L) - d_L - (1+r_L)l_L > 0$, in order to get a positive expected second-period profit. Then, since $f(l_L) - d_L - (1+r_L)l_L > 0$ implies that $f(l_L) - d_L > (1+r_L)l_L \geq 0$, the lender's expected profit π_L, from (1), is $\pi_L = e_L r_L l_L$. The lender's problem is to maximize π_L subject to the *L*-borrower accepting the contract and her rule for choosing e_L as described in Lemma 1. That is, the lender solves the following maximization problem/

Maximize $e_L r_L l_L$

s. t.

$$g'(e_L) = u(f(l_L) - d_L - (1+r_L)l_L) \tag{3}$$

$$f(l_L) - d_L - (1+r_L)l_L > 0 \tag{4}$$

The conditions derived from the Lagrangian for this problem are

$$r_L l_L - \lambda g''(e_L) = 0 \tag{5}$$

$$e_L l_L - \lambda l_L u'(f(l_L) - d_L - (1+r_L)l_L) = 0 \tag{6}$$

$$e_L r_L - \lambda((1+r_L) - f'(l_L))u'(f(l_L) - d_L - (1+r_L)l_L) = 0 \tag{7}$$

where λ is the multiplier associated with the constraint (3), plus constraints (3) and (4). From (6) and (7) we get

$$f'(l_L) = 1 \tag{8}$$

which, by Lemma 2, implies $l_L = l^*$. And (5), (7) and (8) yield

$$e_L g''(e_L) = r_L l_L u'(f(l_L) - d_L - (1+r_L)l_L) \tag{9}$$

Now, with $l_L = l^*$, the problem reduces to finding e_L, r_L which satisfy

$$g'(e_L) = u(f(l^*) - d_L - (1+r_L)l^*) \tag{3'}$$

$$e_L g''(e_L) = r_L l^* u'(f(l^*) - d_L - (1+r_L)l^*) \tag{9'}$$

$$f(l^*) - d_L - (1+r_L)l^* > 0 \tag{4'}$$

Writing $A \equiv f(l^*) - l^* - d_L$, (3'), (9'), (4') become respectively

$$g'(e_L) = u(A - r_L l^*) \tag{3''}$$

$$e_L g''(e_L) = r_L l^* u'(A - r_L l^*) \tag{9''}$$

$$A - r_L l^* > 0 \qquad (4'')$$

Note that $A > 0$, since $f(l^*) - l^* - d_L = h^* - d_L > 0$ by assumption. For use in later analyses we list below the corresponding constraints for the *H*-borrower:

$$g'(e_H) = u(f(l^*) - d_H - (1 + r_H)l^*) \qquad (3^*)$$

$$e_H g''(e_H) = r_H l\ u'(f(l^*) - d_H - (1 + r_H)l^*) \qquad (9^*)$$

$$f(l^*) - d_H - (1 + r_H)l^* > 0 \qquad (4^*)$$

Proposition 1. *There exists a unique solution $(\bar{r}_L, \bar{e}_L)$ that satisfies (3''), (9'') and (4'').*

Proof. It's clear that for each $0 \le r_L \le \frac{A}{l}$, there is a unique $e_L \in [0,1)$ that satisfies (3''). Thus (3'') uniquely defines e_L as a function of r_L: $e_L = p(r_L)$ for $0 \le r_L \le \frac{A}{l}$. It is easily seen that p is a strictly decreasing function and is continuous, with $p(0) = g'^{-1}(u(A)) > 0$ and $p(\frac{A}{l}) = 0$.

In the same way, (9'') uniquely defines e_L as a function of r_L: $e_L = q(r_L)$.Then q is strictly increasing and continuous, with $q(0) = 0$ and $q(\frac{A}{l}) > 0$, since $u' > 0$ and $e_L g''(e_L)$ is increasing in e_L because $g''' > 0$.

Thus finding a solution to (3''), (9'') and (4'') reduces to finding (r_L, e_L) that satisfies $e_L = p(r_L)$, $e_L = q(r_L)$ as well as (4''). Consider $s(r_L) \equiv p(r_L) - q(r_L)$. Then s is continuous and decreasing in r_L, $s(0) = p(0) - q(0) > 0$ and $s(\frac{A}{l^*}) = p(\frac{A}{l^*}) - q(\frac{A}{l^*}) < 0$. By the intermediate-value theorem, there exists a unique $\bar{r}_L$ such that $0 < \bar{r}_L < \frac{A}{l^*}$ and $s(\bar{r}_L) = 0$, or equivalently, $p(\bar{r}_L) = q(\bar{r}_L)$. Let $\bar{e}_L \equiv p(\bar{r}_L) = q(\bar{r}_L) > 0$. Then $(\bar{r}_L, \bar{e}_L)$ is indeed the unique solution to (3'') and (9''). Since $0 < \bar{r}_L < \frac{A}{l^*}$, this solution satisfies (4'') also. Q.E.D.

We can now state the theorem about the SPE for the case $d_L < h^* \le d_H$.

Theorem 2. *If $d_L < h^* \le d_H$, then there is a unique SPE outcome of the game in which the lender offers the H-borrowers no contract and offers the L-borrowers a contract (l_L, r_L) where $l_L = l^*$, and $r_L = r_L^*$, the unique solution to the equations (3') and (9'); the contract is accepted by the L-borrowers; the L-borrowers receive positive expected utility and the lender earns a positive expected profit in the second period.*

Case 3: $d_H < h^*$. Then $d_i < h^*, i = L, H$. Since the lender can perfectly discriminate between the two borrower types, his maximization problem consists of maximizing his expected second-period profit from each borrower type. Clearly, the analysis for Case 2 carries over to the present case so we can establish the following theorem.

Theorem 3. *If $d_H < h^*$, then there is a unique SPE outcome of the game in which, for $i = L, H$, the lender offers the i-borrowers a contract (l_i, r_i), where $l_i = l^*$, and $r_i = r_i^*$, the unique solution to (3') and (9') or (3*) and (9*) depending on whether i = L or H; the two contracts are accepted by the respective borrower types; the i-borrowers receive positive expected utility and the lender earns a positive expected profit in the second period.*

4. Self-selection of Borrowers

In this section we investigate the screening problem when the borrowers' pre-existing debts are unobservable to the lender. Not knowing to which type a borrower belongs, the lender's best guess is that the borrower is as likely an *L*-borrower as an *H*-borrower[8]. A pure strategy for the lender consists of choosing two loan contacts (l', r') and (l'', r''), to be offered simultaneously to the borrowers. A pure strategy for a borrower, when facing the two contracts from the lender, is to decide whether to accept (l', r'), (l'', r''), or neither. If the borrower accepts one of the two contracts, she also needs to choose a corresponding effort level as part of her pure strategy. We adopt a tie-breaking rule that if a borrower is indifferent between two acceptable contracts she always accepts the one with the larger loan size. In this way, the borrowers self-select the lender's contract offers.

The three cases in the last section are again considered in turn, in our search for the SPE under asymmetric information. As it turns out, Cases 1 and 2

[8] An equivalent interpretation of this assumption is that half of the borrower population are *L*-borrowers and another half are *H*-borrowers.

yield essentially the same SPE outcomes as those described in Theorems 1 and 2 respectively. To see this, consider for example Case 2 where $d_L < h^* \le d_H$. Although he cannot observe the actual pre-existing debts of the borrowers, realizing that no contract will be accepted by the *H*-borrower, the lender's maximization problem in the current situation of asymmetric information is essentially the same as that in Section 3 for the same case $d_L < h^* \le d_H$. More precisely, there is a unique SPE outcome of the game, in which the lender offers all borrowers the same two contracts, (l', r') and (l'', r''), where $l' = l^*$, and $r' = r_L^*$, the unique solution to (3') and (9'), and (l'', r'') is some arbitrary loan contract satisfying $f(l'') - d_L - (1 + r'')l'' \le 0$; the *L*-borrower accepts the contract (l', r') and the *H*-borrower accepts neither (l', r') nor (l'', r''); and the second-period payoffs to the two parties are the same as those in Theorem 2.

Therefore, $d_H < h^*$ it is the more interesting case which might give rise to different SPE outcomes than those in Theorem 3, under the current condition of asymmetric information about the borrowers' pre-existing debts. We henceforth restrict our attention to this case.

4.1. Non-existence of Fully-separating Equlibria

Assume $d_H < h^*$. We first partition the lender's strategy space, $D = \{((l', r'), (l'', r''))\}$, into five disjoint subsets: $D = \bigcup_{j=1}^{5} D_j$, where $D_1 = \{((l', r'), (l'', r''))\,|$. The *H*-borrower accepts one of the two contracts and the *L*-borrower accepts neither}, $D_2 = \{((l', r'), (l'', r''))\,|$. The *L*-borrower accepts one of the two contracts, the *H*-borrower accepts the other, and $(l', r') \ne (l'', r'')\}$, $D_3 = \{((l', r'), (l'', r''))\,|$. The *L*-borrower accepts one of the two contracts and the *H*-borrower accepts neither}, $D_4 = \{((l', r'), (l'', r''))\,|$. The *L*-borrower accepts neither and the *H*-borrower accepts neither}, $D_5 = \{((l', r'), (l'', r''))\,|$. The two borrower types accept the same one contract}.

Definition. An SPE of the screening game is called *fully-separating* if the lender's strategy in this SPE lies in D_2; it's called *semi-separating* if the lender's strategy lies in $D_1 \cup D_3$; and it's called *pooling* if the lender's strategy lies in D_5.

We show that some of the D_j 's are actually empty and thus narrow down the search area for the SPE of the screening game.

Proposition 2. *D_1 is empty.*

Proof. Suppose D_1 is not empty. Let $((l^{'},r^{'}),(l^{''},r^{''}))\in D_1$. Without loss of generality suppose the H-borrower accepts $(l^{'},r^{'})$. By Lemma 1, it must be the case that $f(l^{'})-d_H-(1+r^{'})l^{'}>0$. But then $f(l^{'})-d_L-(1+r^{'})l^{'}>0$, which, by Lemma 1, implies that the L-borrower also accepts $(l^{'},r^{'})$. Hence $((l^{'},r^{'}),(l^{''},r^{''}))\notin D_1$, a contradiction. Q.E.D.

Proposition 2 implies that the only possible semi-separating SPE are those in which the lender's strategy lies in D_3. We next show that D_2 is also empty. The proof makes use of the following lemma.

Lemma 3. *Let $t_i(e)\equiv eB_i-g(e), i=1,2$, be two functions defined on $0\le e<1$. Denote by T_i their corresponding maximum value. Then the following holds: for $B_i\in(0,\infty)$, $T_1>T_2$ if and only if $B_1>B_2$; and hence $T_1=T_2$ if and only if $B_1=B_2$.*

Proof. *See Appendix.*

Proposition 3. *D_2 is empty.*

Proof. Suppose D_2 is not empty. Let $((l^{'},r^{'}),(l^{''},r^{''}))\in D_2$. Without loss of generality suppose the L-borrower accepts $(l^{'},r^{'})$ and the H-borrower accepts $(l^{''},r^{''})$. It follows by Lemma 1 that

$$f(l^{'})-d_L-(1+r^{'})l^{'}>0 \tag{10}$$

$$f(l^{''})-d_H-(1+r^{''})l^{''}>0 \tag{11}$$

Let e_L, e_H be, respectively, the corresponding effort choice of the L-borrower and the H-borrower. Then

$$v_L=e_Lu(f(l^{'})-d_L-(1+r^{'})l^{'})-g(e_L),$$

$$v_H=e_Hu(f(l^{''})-d_H-(1+r^{''})l^{''})-g(e_H)$$

are the second-period expected utility for the L-borrower and the H-borrower respectively.

Now consider what the L-borrower's expected utility would be if she, instead of (l',r'), were to choose (l'',r''). Let e_{LH} be her corresponding effort level choice. Then the L-borrower's second-period expected utility is given by

$$v_{LH} = e_{LH}u(f(l'') - d_L - (1+r'')l'') - g(e_{LH}).$$

Similarly, if the H-borrower were to choose (l',r') instead of (l'',r''), her second-period expected utility would be

$$v_{HL} = e_{HL}u(f(l') - d_H - (1+r')l') - g(e_{HL}),$$

where e_{HL} is her corresponding effort level choice when she chooses the contract (l',r').

The fact that the L-borrower accepts (l',r') rather than (l'',r'') means that either $v_L > v_{LH}$, or $v_L = v_{LH}$ and $l'' < l'$ (recall the tie-breaking rule for the borrowers' acceptance of loan contracts), which in turn means either

$$f(l') - d_L - (1+r')l' > f(l'') - d_L - (1+r'')l'' \qquad (L1)$$

or

$$f(l') - d_L - (1+r')l' = f(l'') - d_L - (1+r'')l'' \qquad (L2)$$

and $l'' < l'$, by (10), Lemma 3, and the fact that u is a strictly increasing function.

Similarly, the fact that the H-borrower accepts (l'',r'') rather than (l',r') means either

$$f(l'') - d_H - (1+r'')l'' > f(l') - d_H - (1+r')l' \qquad (H1)$$

or

$$f(l'') - d_H - (1+r'')l'' = f(l') - d_H - (1+r')l' \qquad (H2)$$

and $l' < l''$.

It is a straightforward exercise to show: (L1) and (H1) together yield a contradiction, so do (L1) and (H2) together, (L2) and (H1) together, and (L2) and (H2) together. These contradictions complete the proof of the proposition. Q.E.D.

Strictly speaking, we need to address the following question for the sake of completeness: which contract will a borrower accept if she is indifferent between two different contracts yet the two contracts have the same loan size? It can be easily shown that the borrower *cannot* be indifferent between two different contracts with the same loan size, given that both contracts are acceptable to her.

A corollary to Proposition 3 is that a fully-separating equilibrium does not exist, which we state as a theorem.

Theorem 4. *Assume $d_H < h$. In the contracting game in which the borrowers' pre-existing debts are unobservable to the lender and the lender screens the borrowers, there exists no SPE that is fully-separating.*

4.2. Pooling and Semi-separating Equlibria

Now that D_1 and D_2 are empty we may in the search for an SPE restrict our attention to $D_3 \cup D_4 \cup D_5$. However, an SPE cannot exist in which the lender's strategy is taken from D_4 , for by Theorem 2 he can do strictly better (i.e., earn a positive vs. zero expected profit in the second period) by deviating to a strategy in D_3 (D_3 is not empty since any contract pair $((l^{'},r^{'}),(l^{''},r^{''}))$ satisfying $f(l^{'}) - d_L - (1+r^{'})l^{'} > 0$, $f(l^{'}) - d_H - (1+r^{'})l^{'} \le 0$ and $f(l^{''}) - d_H - (1+r^{''})l^{''} \le 0$ is in D_3). Thus the search area for an SPE is further reduced down to $D_3 \cup D_5$, from which the lender chooses his best strategy.

In what follows we examine the conditions under which a pooling or semi-separating SPE exists. For expositional convenience we collect below some of the objective functions, constraints, optimization problems and other quantities to be referenced frequently in later analyses.

Objective functions:

$$\varphi(l,r,e_L,e_H) \equiv \frac{1}{2}e_L rl + \frac{1}{2}e_H rl ,$$

$$\varphi(l,r,e_L) \equiv \frac{1}{2}e_L rl .$$

Constraints:

$$g^{'}(e_L) = u(f(l) - d_L - (1+r)l) \tag{12}$$

$$g^{'}(e_H) = u(f(l) - d_H - (1+r)l) \tag{13}$$

$$f(l) - d_H - (1+r)l > 0 \tag{14}$$

$$f(l) - d_H - (1+r)l \ge 0 \tag{15}$$

$$f(l) - d_H - (1+r)l \le 0 \tag{16}$$

$$f(l) - d_L - (1+r)l > 0 \tag{17}$$

$$f(l) - d_H - (1+r)l = 0 \tag{18}$$

Optimization problems:

Maximize $\varphi(l,r,e_L,e_H)$ *s.t. (12), (13) and (14),* (P1)

Maximize $\varphi(l,r,e_L)$ *s.t. (12), (16) and (17),* (P2)

Maximize $\varphi(l,r,e_L,e_H)$ *s.t. (12), (13) and (15),* (P3)

Maximize $\varphi(l,r,e_L,e_H)$ *s.t. (12), (13) and (18),* (P4)

Maximize $\varphi(l,r,e_L)$ *s.t. (12) and (18),* (P5)

Maximize $\varphi(l,r,e_L)$ *s.t. (12).* (P6)

Quantities:

l^* is as defined in Lemma 1; e_L^*, r_L^* are the unique solution to the equations (3') and (9'); e_H^*, r_H^* are the unique solution to the equations (3*) and (9*); r^{**} is the unique optimum solution to problem (P1) as given in the following Lemma 4.

We also use the following notation: [Maximize $\Omega(x)$ s.t. (1), (2),…,(n)] denotes the maximum *value* of the corresponding maximization problem if an optimum solution to this problem exists; similarly for the notation [(P)] where (P) is the name of a maximization problem.

Lemma 4. *Assume* $d_H < h^*$. *The maximization problem (P1) has a unique optimum solution* $(l^*, r^{**}, e_L^{**}, e_H^{**})$, *where* l^* *is as defined in Lemma 2, and* r^{**} *satisfies* $\min\{r_L^*, r_H^*\} \le r^{**} \le \max\{r_L^*, r_H^*\}$.

Proof. *See Appendix.*

With these preparations we are now in a position to characterize the pooling and semi-separating SPE of the screening game. This is accomplished in Theorems 5 and 6. Recall that we are searching for the lender's best strategy in the set $D_3 \cup D_5$. Note also that since $f(l^*) - d_H - (1+\max\{r_L^*, r_H^*\})l^* > 0$ implies $d_H < h^*$, the latter condition is not imposed in Theorem 5, while it is needed in Theorem 6 because it is not implied by the condition in Theorem 6, namely, $f(l^*) - d_L - (1+\min\{r_L^*, r_H^*\})l^* \le 0$.

Theorem 5. *If* $f(l^*) - d_H - (1+\max\{r_L^*, r_H^*\})l^* > 0$*, then there is a unique SPE outcome of the screening game, which is pooling. In the SPE, the lender offers both borrower types the same two contracts* (l', r') *and* (l'', r'')*, where* $(l', r') = (l^*, r^{**})$ *and* (l'', r'') *is such that* $f(l'') - d_L - (1+r'')l'' \le 0$*; both borrower types accept the contract* (l', r')*; both borrower types receive positive expected utility and the lender earns a positive expected profit in the second period.*

Proof. Consider first what is the best the lender can do, if his strategy choice is restricted to those in D_5. Since, in D_5, the two borrower types accept the same one contract, the lender need not worry about the other contract in his offer that is not accepted; he can make sure neither borrower type will accept the other contract (l'', r'') by making $f(l'') - d_L - (1+r'')l'' \le 0$. Hence, noting that $f(l) - d_H - (1+r)l > 0$ implies $f(l) - d_L - (1+r)l > 0$, the lender's objective essentially is to solve problem (P1), if his strategy choice is restricted to D_5. Now, consider what the best the lender can do, if his strategy choice is restricted to D_3. Again, the lender can just concentrate on the contract that is accepted by the *L*-borrower, by choosing the other contract (l'', r'') to satisfy $f(l'') - d_L - (1+r'')l'' \le 0$ so that no borrower type will accept this contract. So, essentially, the lender's objective is to solve problem (P2), if his strategy choice is restricted to D_3.

Under the following condition of the Theorem:

$$f(l^*) - d_H - (1+\max\{r_L^*, r_H^*\})l^* > 0 \tag{19}$$

problem (P1) has a unique optimum solution as described in Lemma 4, and thus the value [(P1)] does exist. Consider the 4-tuple $(l^*, r_L^*, e_L^*, \hat{e}_H)$ where $\hat{e}_H$ is given by $g'(\hat{e}_H) = u(f(l\ \ ) - d_H - (1+r_L\ \ )l\ \ )$. Then $\hat{e}_H > 0$ by (19). It can be easily verified that, under (19), the above 4-tuple satisfy (12), (13) and (14). We thus get

$$\phi(l^*, r_L^*, e_L^*, \hat{e}_H) = \frac{1}{2} e_L^* r_L^* l^* + \frac{1}{2} \hat{e}_H r_L^* l^* > \frac{1}{2} e_L^* r_L^* l^*,$$

and hence

$$[(\text{P1})] \ge \phi(l^*, r_L^*, e_L^*, \hat{e}_H) > \frac{1}{2} e_L^* r_L^* l^* \tag{20}$$

On the other hand, we have, for any (l, r, e_L) satisfying (12), (16) and (17), that

$$\varphi(l,r,e_L) \leq \text{[Maximize } \varphi(l,r,e_L) \text{ s.t. (12)]} = \frac{1}{2}e_L^* r_L^* l^* \tag{21}$$

Here, the inequality in (21) results from dropping constraints (16) and (17) from problem (P2). It follows from (20) and (21) that, for any (l,r,e_L) satisfying (12), (16) and (17), $\varphi(l,r,e_L) <$ [(P1)]. Hence, the lender's best strategy lies in D_5 and in D_5 only. Lemma 4 guarantees the existence of and gives a pooling SPE. Q.E.D.

Theorem 6. *Assume $d_H < h^*$. If $f(l^*) - d_L - (1+\min\{r_L^*, r_H^*\})l^* \leq 0$, then there is a unique SPE outcome of the screening game, which is semi-separating. In the SPE, the lender offers both borrower types the same two contracts (l',r') and (l'',r''), where $l' = l^*, r' = \frac{h^* - d_H}{l^*}$, and (l'',r'') is such that $f(l'') - d_L - (1+r'')l'' \leq 0$; the L-borrower accepts the contract (l',r') and the H-borrower accepts neither contract; the L-borrower receives positive expected utility and the lender earns a positive expected profit in the second period.*

Proof. Again we consider the two maximization problems (P1) and (P2), under the condition of the current Theorem:

$$f(l^*) - d_L - (1+\min\{r_L^*, r_H^*\})l^* \leq 0 \tag{22}$$

Consider (P1) first. If (P1) had an optimum solution, then the optimum solution should be $(l^*, r^{**}, e_L^{**}, e_H^{**})$ as given in Lemma 3. In particular it should satisfy (14), i. e.,

$$f(l^*) - d_H - (1+r^{**})l^* > 0 \tag{23}$$

But (23) contradicts (22), since $\min\{r_L^*, r_H^*\} \leq r^{**}$ and $d_L < d_H$. Therefore (P1) does not have an optimum solution under (22).

Now consider problem (P3), which is the same as (P1) except that the constraint (14) is replaced by (15). Since (P1) does not have an optimum solution, constraint (15) must be binding at any optimum solution of (P3). Thus (P3) is equivalent to (P4). For (P4), constraints (13) and (18) imply $e_H = 0$. Since $\varphi(l,r,e_L,0) = \phi(l,r,e_L)$, we have

$$\text{[(P3)] = [(P4)] = [Maximize } \phi(l,r,e_L,0) \text{ s.t. (12) and (18)] = [(P5)]} \tag{24}$$

Because, as shown above, (P1) doesn't have an optimum solution, it must be the case that, for any (l,r,e_L,e_H) satisfying (12), (13) and (14),

$$\phi(l,r,e_L,e_H) < [(\text{P3})] \qquad (25)$$

And (24), (25) imply that, for any (l,r,e_L,e_H) satisfying (12), (13) and (14),

$$\phi(l,r,e_L,e_H) < [(\text{P5})] \qquad (26)$$

Next consider (P2). The optimum solution to (P6), (l^*,r_L^*), doesn't satisfy (17) because $f(l^*)-d_L-(1+r_L^*)l^* \le 0$ by (22). Thus (16) must be binding at any optimum solution of (P2). But once (16) is binding (17) becomes extraneous. Hence, (P2) is equivalent to (P5). It can be easily shown that (P5) does indeed have an optimum solution $(l^*,\tilde{r}_L,\tilde{e}_L)$ where $\tilde{r}_L = \frac{h^*-d_H}{l^*}$ and $\tilde{e}_L = g'^{-1}(u(d_H-d_L))$. This optimum solution of (P5) is also the optimum solution of (P2), and hence

$$[(\text{P5})] = [(\text{P2})]. \qquad (27)$$

To summarize, we have shown, for any (l,r,e_L,e_H) satisfying (12), (13) and (14), that $\phi(l,r,e_L,e_H) < [(\text{P2})]$, by (26) and (27). From this we see that, under (22), the lender's best strategy lies in D_3 and in D_3 only, and is given by $(l^*,\tilde{r}_L)$ as defined above. This leads to the semi-separating SPE outcome as described in the present Theorem. Q.E.D.

5. Credit Rationing and Credit Forcing

In this section we investigate the implications of Theorems 5 and 6 for credit rationing. As mentioned in the Introduction, with variable loan size type I credit rationing becomes a possibility. Clearly, the result of Theorem 5 implies no type II credit rationing because both borrower types receive a loan of size l^* at the interest rate r^{**}. A natural question, then, is whether at the quoted interest rate the loan size is a desirable one from the borrowers' perspective and, if not, in what direction it is distorted.

An *i*-borrower's expected second-period utility, when accepting the loan contract (l_i,r_i), is given by $E_i^{\max}$, the maximum value of

$$E_i = e_i \cdot u(f(l_i)-d_i-(1+r_i)l_i)-g(e_i).$$

By Lemma 4, $E_i^{\max}$ is an increasing function of $u(f(l_i)-d_i-(1+r_i)l_i)$, which in turn implies that it is an increasing function of $f(l_i)-d_i-(1+r_i)l_i$, since

u itself is increasing in wealth. Thus, the most desirable loan size for an i-borrower, at interest rate $r_i = r^{**}$, is the one which maximizes $f(l_i) - d_i - (1+r^{**})l_i$. Denote it by $\tilde{l}_i$. Then $\tilde{l}_i$ satisfies the first-order condition $f'(\tilde{l}_i) = 1 + r^{**}$. From Lemma 2, l^*, the loan size granted to the borrower, is such that $f'(l^*) = 1$, from which we see $\tilde{l}_i > l^*$ because $f'' < 0$. Therefore, the loan size the borrower obtains from the lender exceeds his most desirable size and, instead of type I credit rationing, *credit forcing* arises in this situation, i. e., with only two contract offers to choose from the borrower is «forced» to accept a loan that is larger than her optimal level of investment given interest rate. Since by our assumption about the borrower's production technology her effort level does not depend on the size of input, it should be clear that credit forcing arises here not because a larger loan entails more effort, but rather it is due to the decreasing-returns nature of the production technology.

Credit forcing also occurs with the L-borrower in Theorem 6; at the interest rate $r' = \dfrac{h^* - d_H}{l^*}$ she would like to have a loan smaller in size than l^* but is «forced» to accept a larger one. The H-borrower, on the other hand, suffers from type II credit rationing: the loans available from the lender come at unacceptable terms, and as a result she opts out of the market. The situation here conforms exactly to the definition of type II credit rationing: among a population of observationally indistinguishable borrowers from the lender's point of view, some obtain a loan from the lender while others do not. This, of course, is a consequence of the asymmetric information between the lender and the borrower. It is most clearly seen by comparing the current situation with that in Theorem 3 where, for the same range of parameter values (i. e., $d_H < h^*$ and $f(l^*) - d_L - (1 + \min\{r_L^*, r_H^*\})l^* \le 0$) and without informational asymmetry, the H-borrower would have obtained a loan offered by the lender. It is worth mentioning that the situation in Theorem 2 does not constitute a case of type II rationing *per se* because, although there is complete exclusion of the H-borrower from the market, the two borrower types are nevertheless distinguishable from the lender's point of view.

At this point it is appropriate to contrast our results with those in DeMeza and Webb (1992). In an environment with symmetric information and without moral hazard, they show that a type I credit rationing (or forcing) outcome might be socially efficient. In comparison, consider the credit forcing outcome that occurs in our environment. Although this outcome may not be socially efficient in the first-best sense, it nevertheless leads to a constrained optimum level of investment. This can be seen from the fact that with unobservable debts the loan size is the same as that with observable debts (both at l^*). That l^* may not be the socially efficient level of investment is, of course, due to the unobservability of borrowers' effort choice and the accompanying moral hazard activity which is absent in the environment of DeMeza and Webb. Thus one may call the phenomena in Theorems 5 and 6 «efficient» credit forcing though the term efficient should be interpreted in a constrained (i.e. second-best) sense.

Besides the efficiency issue our results and those in DeMeza and Webb (1992) stand in contrast on two other issues. They show that credit rationing is impossible with a monopoly lender. Of course, with their exclusive focus on type I credit rationing; type II rationing is left out of the picture. In contrast, our analysis shows that with asymmetric information type II credit rationing could well be a possibility. However, if we return to the environment of symmetric information (Theorem 3), then type I credit rationing does disappear and instead there is credit forcing by the monopoly lender. They also claim that, in their environment of competitive lenders, if the lenders cannot observe borrowers' total indebtedness or ensure the priority rule for debt repayment there is no point in an individual lender rationing credit. In our environment of a monopolist lender, however, it is exactly because of these two constraints that cause the lender to ration credit[9].

6. Concluding Remarks

In this paper we make a first step in the literature to explore a hybrid model of credit rationing with multidimensional incentive structure. For a range of different parameter values we fully characterize the subgame perfect equilibria of the loan contracting game. Under certain parameter values there is type II credit rationing for one borrower type and credit forcing for another. Credit forcing is shown to be efficient in a constrained sense. By the standard of the existing literature on the subject our model is a fairly general one with multi-dimensional incentive structure, endogenous project risk and variable loan size.

In future research it might be desirable to study a model in a competitive environment with essentially the same features as the present one. As indicated by the analysis in Gale and Hellwig (1985) the divide between a monopolistic setting and a competitive one may not be as stark as it appears, as far as deriving the optimal contract is concerned. Indeed, by incorporating an additional zero-profit condition for the lender into the constraints of the corresponding maximization problem, optimal contract can be derived in much the same way as it is for the monopoly case. The real challenge lies in the game-theoretic formulation of competition in contracts when there is multi-dimensional informational asymmetry, as the anomalies (e. g., the non-existence of equilibrium) in Hellwig (1987)

9 On this issue our result is similar to that of Longhofer (1997) who investigates how absolute priority rule violations in financial markets cause or exacerbate credit rationing.

would suggest. The issue is further complicated by the findings of Parlour and Rajan (2001) which call into question even the zero-profit assumption in a competitive credit market. We certainly do not expect the task of game-theoretic modeling in such an environment to be an easy one.

Bibliography

1. Besanko, D. and A. Thakor (1987). Collateral and rationing: Sorting equilibria in monopolistic and competitive credit markets. *International Economic Review,* 28, 671–690.
2. Bester, H. (1985a). Screening vs. rationing in credit markets with imperfect information. *American Economic Review,* 75, 850–55.
3. Bester, H. (1985b). The level of investment in credit markets with imperfect information. *Journal of Institutional and Theoretical Economics,* 141, 503–515.
4. Bester, H. and M. Hellwig (1987). Moral hazard and equilibrium credit rationing. In: *Agency Theory, Information and Incentives*, edited by G. Bamberg and K. Spremann.
5. Bisin, A. and D. Guaitoli (2004). Moral hazard and nonexclusive contracts. *RAND Journal of Economics,* 35, 306–328.
6. Bizer, D. and P. DeMarzo (1992). Sequential banking. *Journal of Political Economy,* 100, 41–61.
7. DeMeza, D. and D. Webb (1987). Too much investment: a problem of asymmetric information. *Quarterly Journal of Economics,* 102, 281–292.
8. DeMeza, D. and D. Webb (1992). Efficient credit rationing. *European Economic Review,* 36, 1277–1290.
9. Detragiache, E., P. Garella, and L. Guiso (2000). Multiple versus single banking relationships. *Journal of Finance,* 55, 1133–1161.
10. Gale, D. and M. Hellwig (1985). Incentive compatible debt contracts: The one-period problem. *Review of Economic Studies,* LII, 647-664.
11. Grinblatt, M. and C.Y. Hwang (1989). Signaling and the pricing of new issues. *Journal of Finance,* 46, 393–420.
12. Hellmann, T. and J. Stiglitz (2000). Credit and equity rationing in markets with adverse selection. *European Economic Review,* 44, 281–304.
13. Hellwig, M. (1987). Some recent developments in the theory of competition in markets with adverse selection. *European Economic Review,* 31, 319–325.

14. Iwasaki, K. (2004, Summer). China's budding credit card market. *Japan Research Quarterly*.
15. Keeton, W. (1979). *Equilibrium Credit Rationing*, Garland Publishing Co.
16. Kletzer, K. (1984). Asymmetries of information and LCD borrowing with sovereign risk. *Economic Journal,* 94, 287–307
17. Li, Yang et al (2005, November). *Dianzi zhifu yu zhongguo jingji.* [Electronic Payments and Chinese Economy], report published by the Institute of Banking and Finance, Chinese Academy of Social Sciences.
18. Longhofer, S. (1997). Absolute priority rule violations, credit rationing, and efficiency. *Journal of Financial Intermediation,* 6, 249–267.
19. Longhofer, S. and C. Carlstrom (1995). Absolute priority rule violations in bankruptcy. Federal Reserve Bank of Cleveland, *Economic Review,* 31, 21–30.
20. Milde, H. and J. Riley (1988). Signaling in credit markets. *Quarterly Journal of Economics,* 103, 101–129.
21. Pagano, M. and T. Jappelli (1993). Information sharing in credit markets. *Journal of Finance,* 48, 1693–1718.
22. Park, I.U. (2004). Moral hazard contracting and private credit markets. *Econometrica,* 72, 701–746.
23. Parlour, C. and U. Rajan (2001). Competition in loan contracts. *American Economic Review,* 91, 1311–1328.
24. Petersen, M. and R. Rajan (1994). The benefits of lending relationships: evidence from small business data. *Journal of Finance,* 49, 3–37.
25. Petersen, M. and R. Rajan (1995). The effect of credit market competition on lending relationships. *Quarterly Journal of Economics,* 110, 407–444.
26. Stiglitz, J. and A. Weiss (1981). Credit rationing in markets with imperfect information. *American Economic Review,* 71, 393–409.
27. Townsend, R. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory,* 21, 265–293.
28. Wette, H. (1983). Collateral in credit rationing in markets with imperfect information: Note. *American Economic Review,* 73, 442–445.
29. Williamson, S. (1986). Costly monitoring, financial intermediation, and equilibrium credit rationing. *Journal of Monetary Economics,* 18, 159–179.
30. Williamson, S. (1987). Costly monitoring, loan contracts, and equilibrium credit rationing. *Quarterly Journal of Economics,* 102, 135–145.

Appendix

Proof of Lemma 1

For sufficiency, it suffices to show: the borrower's maximum expected utility by accepting the loan contract (l_i, r_i) is strictly positive if $f(l_i) - d_i - (1+r_i)l_i > 0$.

If the borrower accepts the contract (l_i, r_i) and chooses an effort level e_i, then her expected utility, under limited liability, is given by

$$E_i = E[u(\max\{f(l_i)Z(e_i) - d_i - (1+r_i)l_i, 0\}) - g(e_i) \mid e_i]$$
$$= e_i \cdot u(\max\{f(l_i) - d_i - (1+r_i)l_i, 0\}) - g(e_i).$$

If $f(l_i) - d_i - (1+r_i)l_i > 0$, then $E_i = e_i \cdot u(f(l_i) - d_i - (1+r_i)l_i) - g(e_i)$, and E_i attains its maximum at the solution, $\hat{e}_i$, to the equation

$$g'(e_i) = u(f(l_i) - d_i - (1+r_i)l_i) \tag{A1}$$

with the maximum value

$$E_i^{\max} = \hat{e}_i \cdot u(f(l_i) - d_i - (1+r_i)l_i) - g(\hat{e}_i) \tag{A2}$$

It remains to show: (i) a solution to (A1) exists and is unique, and (ii) $E_i^{\max} > 0$.

Recall the properties of g: $g(0) = 0, g(1) = \infty, g'(0) = 0,$ and $g'' > 0$. It follows immediately that g' is strictly increasing and $g'(e) > 0$ for $0 < e < 1$. We show that g' is unbounded above. Suppose, to the contrary, that $g' \le M$ for some $M > 0$. Then, by the mean-value theorem, for any $0 < e < 1$, there exists $\hat{e}$, with $0 < \hat{e} < e$, such that $g(e) = g(e) - g(0) = g'(\hat{e}) \cdot (e - 0) = g'(\hat{e}) \cdot e.$ But then we have $g(e) = g'(\hat{e}) \cdot e \le M \cdot 1 = M$ for any $0 < e < 1$, contradicting the assumption $g(1) = \infty$. Hence g' is strictly increasing and unbounded above, which implies that (A1) has a unique solution $\hat{e}_i > 0$, by the intermediate-value theorem, the continuity of g', the assumption $g'(0) = 0$, and the fact that $u(f(l_i) - d_i - (1+r_i)l_i) > 0$ (which follows from the properties of u). This proves (i).

To prove (ii), use again the mean-value theorem. By this theorem, there exists $\tilde{e}$, with $0 < \tilde{e} < \hat{e}_i$, such that $g(\hat{e}_i) = g(\hat{e}_i) - g(0) = g'(\tilde{e}) \cdot (\hat{e}_i - 0) = g'(\tilde{e}) \cdot \hat{e}_i$ which, when substituted into (A2), yields

$$E_i^{\max} = \hat{e}_i \cdot (u(f(l_i) - d_i - (1+r_i)l_i) - g'(\hat{e})) = \tilde{e} \cdot (g'(\hat{e}_i) - g'(\tilde{e})) > 0,$$

using (A1) and since $\hat{e}_i > 0$, $\tilde{e} < \hat{e}_i$, and g' is strictly increasing. This proves (ii).

For necessity, note that if $f(l_i) - d_i - (1+r_i)l_i \leq 0$, then $E_i = -g(e_i)$, and the maximum of E_i obtains at $e_i = 0$ with the maximum value 0. The proof is thus complete. Q.E.D.

Proof of Lemma 3

The maximum T_i of $t_i(e) \equiv eB_i - g(e)$ obtains at e_i, the unique solution to equation $t_i'(e) = B_i - g'(e) = 0$, $i = 1,2$. If $B_1 > B_2$, then it follows from $g'' > 0$ that $e_1 > e_2$. By the mean value theorem, there exists $\tilde{e}$, with $e_1 > \tilde{e} > e_2$, such that

$$g(e_1) - g(e_2) = g'(\tilde{e})(e_1 - e_2),$$

which together with $B_1 = g'(e_1)$ and the fact that $g'' > 0$ imply

$$e_1B_1 - e_2B_2 > B_1(e_1 - e_2) > g'(\tilde{e})(e_1 - e_2) = g(e_1) - g(e_2),$$

or,

$$e_1B_1 - g(e_1) > e_2B_2 - g(e_2).$$

Thus we have shown that if $B_1 > B_2$ then $T_1 > T_2$, from which all conclusions of the Lemma follow. Q.E.D.

Proof of Lemma 4

We re-state here the maximization problem (P1):

Maximize $\phi(l, r, e_L, e_H) \equiv \frac{1}{2}e_L rl + \frac{1}{2}e_H rl$

s.t.

$$g'(e_L) = u(f(l) - d_L - (1+r)l) \quad (12)$$

$$g'(e_H) = u(f(l) - d_H - (1+r)l) \quad (13)$$

$$f(l) - d_H - (1+r)l > 0 \quad (14)$$

We are done if we can show the current maximization problem with the equality constraints only (i.e., with constraint (14) dropped) has a unique optimum solution which also satisfies the inequality constraint (14).

Denote by λ_1 and λ_2 the Lagrangian multipliers associated with the constraints (12) and (13) respectively. Then the conditions for the optimization problem are

$$\frac{1}{2}(e_L+e_H)l-\lambda_1 l\cdot u'(f(l)-d_L-(1+r)l)-\lambda_2 l\cdot u'(f(l)-d_H-(1+r)l)=0 \tag{A3}$$

$$\frac{1}{2}(e_L+e_H)r-\lambda_1((1+r)-f'(l))\cdot u'(f(l)-d_L-(1+r)l) \tag{A4}$$

$$-\lambda_2((1+r)-f'(l))\cdot u'(f(l)-d_H-(1+r)l)=0\ ,$$

$$\frac{1}{2}rl-\lambda_1 g''(e_L)=0 \tag{A5}$$

$$\frac{1}{2}rl-\lambda_2 g''(e_H)=0 \tag{A6}$$

together with constraints (12) and (13).

From (A3) and (A4) we get $f'(l)=1$, which by Lemma 2 has a unique solution $l=l^*$. Substitute this into (A3) and we have

$$\frac{1}{2}(e_L+e_H)-\lambda_1\cdot u'(f(l^*)-d_L-(1+r)l^*)-\lambda_2\cdot u'(f(l^*)-d_H-(1+r)l^*)=0 \tag{A7}$$

We showed in the proof of Proposition 1 that with $l=l^*$ (12) uniquely defines e_L as a decreasing function of r :

$$e_L=p_1(r) \tag{A8}$$

and similarly with $l=l$ (13) uniquely defines e_H as a decreasing function of r :

$$e_H=p_2(r) \tag{A9}$$

Now (A7), (A8) and (A9) together with (A5) and (A6) (with $l=l^*$) yield

$$(e_L+e_H)-rl^*[\frac{1}{g''(e_L)}u'(f(l^*)-d_L-(1+r)l^*) \tag{A10}$$

$$+\frac{1}{g''(e_H)}u'(f(l^*)-d_H-(1+r)l^*)]=0$$

We next show that the system of equations (A8), (A9) and (A10) has a unique solution $(r^{**}, e_L^{**}, e_H^{**})$. To do this, substitute (A8) and (A9) into (A10) to get

$$p(r) \equiv p_1(r) + p_2(r) = rl^*[\frac{1}{g''(p_1(r))}u'(f(l^*) - d_L - (1+r)l^*) \tag{A11}$$

$$+\frac{1}{g''(p_2(r))}u'(f(l^*) - d_H - (1+r)l^*)] \equiv q(r)$$

Then, by the properties of g'', u', p_1 and p_2, we see that q is an increasing function of r while p is a decreasing function of r. Recall the quantities, r_L^* and r_H^* and the corresponding e_L^* and e_H^*, defined in Section 4. Without loss of generality suppose $r_L^* < r_H^*$. Also, recall two equations from Section 3:

$$e_L g''(e_L) = r_L l_L u'(f(l_L) - d_L - (1+r_L)l_L) \tag{9}$$

$$e_H g''(e_H) = r_H l^* u'(f(l^*) - d_H - (1+r_H)l^*) \tag{9*}$$

Then we get from the analysis in Section 3 that

$$p_1(r_L^*) = r_L^* l^*[\frac{1}{g''(p_1(r_L^*))}u'(f(l^*) - d_L - (1+r_L^*)l^*)] \tag{A12}$$

and, by the properties of g'', u' and p_2, also that

$$p_2(r_L^*)g''(p_2(r_L^*)) > p_2(r_H^*)g''(p_2(r_H^*)) = r_H^* l^* u'(f(l^*) - d_H - (1+r_H^*)l^*) \tag{A13}$$

$$> r_L^* l^* u'(f(l^*) - d_H - (1+r_L^*)l^*)$$

or equivalently

$$p_2(r_L^*) > r_L^* l^*[\frac{1}{g''(p_2(r_L^*))}u'(f(l^*) - d_H - (1+r_L^*)l^*)] \tag{A14}$$

From (A12) and (A14) it is clear that (cf.(A11))

$$p(r_L^*) > q(r_L^*) \tag{A15}$$

In exactly the same manner we can show

$$p(r_H^*) > q(r_H^*) \tag{A16}$$

From (A15) and (A16), by the intermediate-value theorem and the fact that q is an increasing function of r while p is a decreasing function of r, we see that there exists a unique solution r^{**} to equation (A11). Thus, defining $e_L^{**} = p_1(r^{**})$ and $e_H^{**} = p_2(r^{**})$, we have found a unique solution to the system of equations

(A8), (A9) and (A10). Clearly, $r_L^* \le r^{**} \le r_H^*$. Finally, since it is clear that $e_H^{**} > e_H^* > 0$, it follows from the properties of g' and u that the unique optimum solution to the problem with equality constraints (with $l = l^*$) also satisfies (14). Q.E.D.

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