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Economic Theory

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UNISECTORAL MODEL OF ECONOMIC GROWTH WITH INTELLECTUAL CAPITAL INCLUDED

Abstract

The paper proposed and investigated a new model of optimal growth, which takes into account investments in physical capital and human capital with its additive part, which is the intellectual capital. The existence of the backbone path is proved, and transitional dynamics is studied.

Key words:

Model of economic growth, investment in physical capital, investment in intellectual capital, backbone path, transitional dynamics.

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Introduction. The theory of macroeconomic production functions has been successfully applied to the construction and study of a number of current models of economic growth [1]. Recently, much attention is paid to the optimal control models that include investments in physical and human capitals, particularly in that part which is called the intellectual capital. [2]

The production function of $Y = F(K, L)$, where $K(t)$ physical capital, $L(t)$ labor (human capital), is called neoclassic, if it has the following properties:

1. Constant efficiency under changing the scale of production:

$$F(\lambda K, \lambda L) = \lambda F(K, L) \text{ for all } \lambda > 0 ;$$

2. Positive and diminishing returns of resources:

a) for all $K > 0$ and $L > 0$ is $F(K, L) > 0$,

b) $\frac{\partial F}{\partial K} > 0$, $\frac{\partial^2 F}{\partial K^2} < 0$; $\frac{\partial F}{\partial L} > 0$, $\frac{\partial^2 F}{\partial L^2} < 0$;

3. Inada terms

$$\lim_{K \to 0} \frac{\partial F}{\partial K} = \lim_{L \to 0} \frac{\partial F}{\partial L} = \infty; \ \lim_{K \to \infty} \frac{\partial F}{\partial K} = \lim_{L \to \infty} \frac{\partial F}{\partial L} = 0;$$

4. Materiality:

$$F(0, L) = F(K, 0) = 0 .$$

Problem Statement. Assume, that the resources in the production function are the physical and human capitals, the latter of which additively includes intellectual capital that increases this resource:

$$Y = F(K, \hat{L}), \ \hat{L} = L + H , \quad (1)$$

where $F(\cdot)$ – neoclassical production function, K – physical capital, $\hat{L} = L + H$ – human capital, L – labor, H – intellectual capital, which in this case is measured with additional units of labour force. This approach to represent the production function is new, and is not found in the known classical works [2].

The output can be used for consumption or investment in physical and intellectual capitals. We assume that the amount of physical and intellectual capitals are amortized and withdrawn under frequency of δ_K and δ_L accordingly.

Resource limitation of the economy takes the form of

$$Y = C + I_K + I_H , \tag{2}$$

where I_K and I_H are gross investment into physical and intellectual capitals, respectively. Changes in the two forms of capital are described by differential equations:

$$\begin{aligned} \dot{K} &= I_K - \delta_K K, \;\; K(0) = K_0, \\ \dot{H} &= I_H - \delta_H H, \;\; H(0) = H_0. \end{aligned} \tag{3}$$

Labour L is growing with the known rate of:

$$L(t) = L_0 e^{nt}, \;\; n > 0 . \tag{4}$$

Suppose all the conditions of the neoclassical production function are met for the production function (1) , including Inada conditions, which in our case will take the following form:

$$\lim_{K \to 0} \frac{\partial F}{\partial K} = \lim_{\hat{L} \to 0} \frac{\partial F}{\partial \hat{L}} = \infty; \;\; \lim_{K \to \infty} \frac{\partial F}{\partial K} = \lim_{\hat{L} \to \infty} \frac{\partial F}{\partial \hat{L}} = 0. \tag{5}$$

We believe that households maximize the cumulative utility function

$$U = \int_0^\infty u(c(t)) e^{-\rho t} dt, \;\; \rho > 0 , \tag{6}$$

where

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}, \;\; \theta > 0 , \tag{7}$$

is the utility function with constant inter-temporal elasticity of substitution [2]

$$\sigma = -\frac{u'(c)}{u''(c)c} = \frac{1}{\theta} . \tag{8}$$

Results of the research. The given optimal control problem (1) – (8) will be solved using the Pontriahin maximum principle [4]. For this purpose we will make the Hamiltonian

$$\begin{aligned} J = u(c)e^{-\rho t} &+ \mu(I_K - \delta_K K) + \nu(I_H - \delta_H H) + \\ &+ \omega\left(F\left(K, \hat{L}\right) - C - I_K - I_H\right), \end{aligned} \tag{9}$$

where μ and ν are shadow prices associated with K and H respectively, and ω is a Lagrange multiplier associated with equation (2). At that we note that the restrictions of gross investments non-negativity $I_K \geq 0,\ I_H \geq 0$ so far we do not take into account.

The necessary optimality conditions of the first order are given by:

$$\frac{\partial J}{\partial c} = 0 \quad \Rightarrow \quad u'(c)e^{-\rho t} = \omega,$$

$$\frac{\partial J}{\partial I_K} = 0 \quad \Rightarrow \quad \mu = \omega,$$

$$\frac{\partial J}{\partial I_H} = 0 \quad \Rightarrow \quad \nu = \omega,$$

$$\dot{\mu} = -\frac{\partial J}{\partial K} = 0 \quad \Rightarrow \quad \dot{\mu} = \mu\delta_K - \omega\frac{\partial F}{\partial K},$$

$$\dot{\nu} = -\frac{\partial J}{\partial H} = 0 \quad \Rightarrow \quad \dot{\nu} = \nu\delta_H - \omega\frac{\partial F}{\partial H}.$$

Hence, we obtain $\mu = \nu = \omega$, from where follows

$$\frac{\partial F}{\partial K} - \frac{\partial F}{\partial H} = \delta_K - \delta_H. \tag{10}$$

From the ratio (7) and the expression for the variable ω we obtain

$$\frac{\dot{\omega}}{\omega} = -\theta\frac{\dot{c}}{c} - \rho, \tag{11}$$

whence we obtain the value for the growth rate of consumption

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(-\frac{\dot{\omega}}{\omega} - \rho\right) = \frac{1}{\theta}\left(\frac{\partial F}{\partial K} - \delta_K - \rho\right) = \frac{1}{\theta}\left(\frac{\partial F}{\partial H} - \delta_H - \rho\right). \tag{12}$$

We introduce an auxiliary variable

$$z := \frac{\hat{L}}{K} = \frac{L + H}{K} \tag{13}$$

and consider the ratio (10) as an equation relatively the variable *z*.

From the linear homogeneity of the production function $F(K, \hat{L})$ follows that the values $\frac{\partial F}{\partial K}$ and $\frac{\partial F}{\partial \hat{L}}$ are homogeneous functions of 0 degree, that is, we can write

$$\frac{\partial F}{\partial K}=\phi(z)>0,\ \frac{\partial F}{\partial \hat{L}}=\frac{\partial F}{\partial H}=\psi(z).$$

However,

$$\frac{\partial^2 F}{\partial K^2}=\phi'(z)\left(-\frac{\hat{L}}{K^2}\right)<0 \quad \Rightarrow \quad \phi'(z)>0\,,$$

$$\frac{\partial^2 F}{\partial \hat{L}^2}=\psi'(z)\frac{1}{K}<0 \quad \Rightarrow \quad \psi'(z)<0\,.$$

Using Inada conditions (5), we obtain

$$\lim_{z\to 0}\phi(z)=\lim_{K\to\infty}\frac{\partial F}{\partial K}=0,\ \lim_{z\to 0}\psi(z)=\lim_{\hat{L}\to 0}\frac{\partial F}{\partial \hat{L}}=\infty,$$
$$\lim_{z\to\infty}\phi(z)=\lim_{K\to 0}\frac{\partial F}{\partial K}=\infty,\ \lim_{z\to\infty}\psi(z)=\lim_{\hat{L}\to\infty}\frac{\partial F}{\partial \hat{L}}=0.$$

Then we rewrite equation (10) in the form of

$$\phi(z)-\psi(z)=\delta_K-\delta_L\,. \tag{14}$$

In equation (14) there is the function monotonically increasing from the left, which under the increasing of argument z in the interval $(0,\infty)$ will increase from $-\infty$ to ∞. In this case, it is clear that equation (14) has a unique positive solution $z^* \geq 0$.

The equality of $z=z^*$ is the condition of equality of net marginal product of physical capital and net marginal product of intellectual capital

$$\frac{\partial F^*}{\partial K}-\delta_K=\frac{\partial F^*}{\partial H}-\delta_H\,.$$

This implies that under $z=z^*$ the net rate of return of physical and intellectual capital is equal

$$r^*=\frac{\partial F^*}{\partial K}-\delta_K=\frac{\partial F^*}{\partial H}-\delta_H\,. \tag{15}$$

If the ratio $\frac{\hat{L}}{K}$ is constant, then equation (10) implies that the ratio $\frac{\dot{c}}{c}$ is also constant and after setting $z=z^*$ in (12) we find that

$$\frac{\dot{c}}{c} = \gamma^* = \frac{1}{\theta}\left(\frac{\partial F^*}{\partial K} - \delta_K - \rho\right) = \frac{1}{\theta}\left(\frac{\partial F^*}{\partial H} - \delta_H - \rho\right). \tag{16}$$

At that it is assumed that the model parameters are such that value $\gamma^* > 0$.

To demonstrate how our model corresponds to the general notion, we substitute the expression of $\frac{L+H}{K} = z^*$ into the production function (1) and obtain

$$Y = F(1, z^*)K = AK, \;\; A > 0. \tag{17}$$

So, we see that our model is ultimately equivalent to the classic *AK*-model [2], in which there is no clear separation of different capitals.

Therefore, in our model we can apply the methods of analysis with respect to the *AK* model and show that if the condition of transversality [2] is done, the growth rate of Y, K and $\hat{L}$ should be equal to the *c* growth rate. We note, that the condition of transversality is $r^* > \gamma^*$ [2]. We obtain $\gamma^* = \frac{1}{\theta}(r^* - \rho) > 0$.

Now we will study the item of consideration in this model (1)–(8) additional restrictions for gross investment non-negativity.

Suppose that the economy starts with two volumes of capitals $K(0)$ and $H(0)$. If the ratio $\frac{L(0)+H(0)}{K(0)}$ deviates from the value of z^* found from the equation (14), the optimal solution necessitates a discrete jump in these two volumes so , that it would be immediately reached the value z^*. That is, we have to assume that investments are instantly interchangeable. This is unrealistic. Investors can choose where to invest: either in intellectual capital or in physical. But if the decision is implemented, it is irreversible. Mathematically, these conditions of irreversibility take the form of constraints of inequalities $I_K \geq 0$ та $I_H \geq 0$.

If $\frac{L(0)+H(0)}{K(0)} < z^*$, that is, when *H* at the beginning of time is small relatively *K* our solution implies the increase of *H* and decrease of *K* in zero moment of time. The desire to reduce *K* by the discrete value leads to the fact, that inequality $I_K \geq 0$ is a binder at the initial time. Then the household chooses $I_K = 0$, the growth rate *K* is set by the equation $\frac{\dot{K}}{K} = -\delta_K$, so that the trajectory is defined by the equation.

$$K(t) = K(0)e^{-\delta_K t},\ t > 0 . \qquad (18)$$

The key moment here is that $\frac{L+H}{K}$ increases and reaches the optimum value of z^* in finite time. At this point, the marginal product of human and physical capitals are equal. And the restrictions of gross investment non-negativity in physical capital ceases to be binding. While both types of capital (physical and human) grow with the same rate γ^*, which is determined by equation (16). We have previously suggested that the model parameters are these that $\gamma^* > 0$.

Prior to acquiring the steady state $\frac{L(0)+H(0)}{K(0)} < z^*$ and $I_K = 0$.

If $I_K = 0$ the household optimization problem can be written in the form of a simplified Hamiltonian

$$J = u(c)e^{-\rho t} + \omega\left(F\left(K,\hat{L}\right) - C - \delta_H H\right). \qquad (19)$$

Thus, this model is equivalent to the standard neoclassical growth model in which households choose between consumption and investment in one type of capital *H* in the presence of exogenous technological progress that increases the intensity of the second resource use, in this case of *K*. In the standard model this second resource, i. e. physical capital, grows with constant rate, while in this given model the second resource *K* is growing with negative rate $-\delta_K$

Thus, the dynamic $\hat{L}$ and Y is consistent with the neoclassical growth model. As follows from the analysis of section 2 of the monograph [2], the solution has the property of convergence in the sense that the growth rate of

$$\gamma_{\hat{L}} = \frac{\dot{\hat{L}}}{\hat{L}} \text{ та } \gamma_Y = \frac{\dot{Y}}{Y}$$

monotonically decreases with time. Since these two growth rates monotonically decrease to a value of $\gamma^* > 0$, they should be positive. Consequently, $\frac{L+H}{K}$ is monotonically growing with time, partly due to a decrease of *K* (with δ_K) rate), and partly due to the increase of $\hat{L}$ (with a rate that decreases to γ^*). With $\frac{\hat{L}}{K}$ growth it implies that the net marginal product of human capital, and hence the rate of return, is monotonically decreasing. This trajectory of lowering rate of return corresponds to typical reducing trajectory of γ_c.

It follows from this analysis that the dependence of the production growth rate γ_Y on the value of the $\frac{\hat{L}}{K}$ ratio is the reverse till the time when $\frac{\hat{L}}{K}$ is less than the stationary value of z^*. Dependence of γ_Y upon $\frac{\hat{L}}{K}$ в in [2] is named as the effect of imbalances.

Similar results are obtained when the economy begins to grow in terms of relative abundance of human capital

$$\frac{L(0)+H(0)}{K(0)} > z^* .$$

Conclusions. Thus, this paper proposed a new model of economic growth, which takes into account the physical (*K)* and intellectual (*H*) capitals. The intellectual capital is viewed as the additive part of human capital ($\hat{L} = L + H$). Also the existence of the backbone trajectory is shown. The transient dynamics is analyzed to exit from the initial state to the backbone path.

Bibliography

1. Пономаренко О. І. Сучасний економічний аналіз: У 2 ч. – Ч. 2. Макроекономіка: [навч. посібник] / О. І. Пономаренко, М. О. Перестюк, В. М. Бурим. – К.: Вища школа, 2004. – 207 с.
2. Барро Дж. Экономический рост / Р. Дж. Барро, Х. Сала-и-Мартин: [пер. с англ.] – М.: БИНОМ. Лаборатория знаний, 2010. – 824 с.
3. Inada K. On a Two-Sector Model of Economic Growth: Comments and a Generalization // Review of Economic Studies. – 1963. – Vol. 30. – № 2. – P. 119–127.
4. Григорків В. С. Оптимальне керування в економіці: [навч. посібник] / В. С. Григорків. – Чернівці: Чернівецький нац. ун-т, 2011. – 200 с.

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