JOURNAL
OF EUROPEAN ECONOMY
Vol. 7 (№ 4). December 2008
Publication of Ternopil National Economic University

Macroeconomics

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ON MAIN TRAJECTORY OF NON LINEAR MODEL OF INPUT-OUTPUT ECOLOGICAL ECONOMIC BALANCE EXISTANCE

Abstract

Non linear expansion of classic dynamic model (π-model) on ecological economic system case is proposed. It is outlined in the issue that main trajectory of sustainable development of such kind system exists when non linear model functions are non negative steadily increasing linear homogeneous ones.

Key words:

Ecological economic system, input-output economic ecological model, main trajectory, the Frobenius number, the Frobenius vector, the Neumann ray.

JEL: Q57.

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Non linear model of input-output ecological economic balance was proposed and the question of its non negative solution existence has been investigated in [1]:

$$\begin{array}{l} x^1 = \Phi_{11}(x^1) + \Phi_{12}(x^2) + y^1, \quad y^1 > 0, \\ x^2 = \Phi_{21}(x^1) + \Phi_{22}(x^2) - y^1, \quad y^2 > 0, \end{array} \quad (1)$$

where x^1 – column vector of national product of the main production, y^1 – column vector of final production, x^2 – column vector of pollutants dispose by means of additional production (purifying devices), $\Phi_{11}(x^1)$ – column vector of material production waists on product unit (direct material waists of the main production vector), $\Phi_{12}(x^2)$ – column vector of material production waists in unit pollution damage by purifying devices, $\Phi_{21}(x^1)$ – column vector of pollutants production by means of the main production, $\Phi_{22}(x^2)$ – column vector of pollutants production by means of additional production (pollutants dispose).

Sufficient condition of non negative solution existence and uniqueness $x = (x^1, x^2)^T$ obtains the form [1]:

$$\Phi_{21}(y^1) \geq y^2. \quad (2)$$

Dynamic linear model of input-output ecological economic balance model was proposed and the subject of main developments existence and uniqueness has been investigated [2]. This is expanded version of classic π-model [3]. This article deals with the following expansion which means the transition of dynamic input-output model of ecological economic balance on non linear case.

It is proposed the following model of optimal ecologic economic development:

$$\begin{array}{l} f_1(x_T^1) + f_2(x_T^2) \to \max, \\ \Phi_{11}(x_t^1) + \Phi_{12}(x_t^2) + D_{11}(\eta_t^1) + D_{12}(\eta_t^2) + c^1 L_t \leq x_t^1, \\ \Phi_{12}(x_t^1) + \Phi_{22}(x_t^2) + D_{21}(\eta_t^1) + D_{22}(\eta_t^2) + c^2 L_t \leq x_t^1 + y_t^2, \\ x_t^1 \leq \xi_{t-1}^1, \; x_t^2 \leq \xi_{t-1}^2, \\ \xi_t^1 \leq \xi_{t-1}^1 + \eta_t^1, \; \xi_t^2 \leq \xi_{t-1}^2 + \eta_t^2, \\ l^1(x_t^1) + l^2(x_t^2) \leq L_t, \\ y_t^2 \leq H_1(x_t^1) + H_2(x_t^2), \\ x_t^1 \geq 0, x_t^2 \geq 0, \xi_t^1 \geq 0, \xi_t^2 \geq 0, \eta_t^1 \geq 0, \eta_t^2 \geq 0, L_t \geq 0, y_t^2 \geq 0 \end{array} \quad (3)$$

where $\xi_0^1 > 0$ and $\xi_0^2 > 0$ are given vectors.

In model (3): x_t^1 – vector of full production in the period t; x_t^2 – vector of pollutants utilized by purifying devices; y_t^2 – vector of pollutants in the environment (indisposed pollutants); ξ_t^1 – vector of power in production; ξ_t^2 – vector of power in pollutants dispose; η_t^1 – vector of power increase of producing industries; η_t^2 – vector of power increase of purifying devices; $\Phi_{11}(x_t^1)$ – vector of material wastes in producing by basic production in x_t^1; $\Phi_{12}(x_t^2)$ – vector of material wastes in purifying devices in case of pollutants utilization in x_t^2; $\Phi_{21}(x_t^1)$ – vector of pollutants production in case of production x_t^1; $\Phi_{22}(x_t^2)$ – vector of the second pollutants production by purifying devices in case of pollutants utilization in x_t^2; $D_{11}(\eta_t^1)$ – vector of material wastes in additional increase of basic production power construction in η_t^1; $D_{12}(\eta_t^2)$ – vector of material wastes in additional increase power of purifying devices construction η_t^2; $D_{21}(\eta_t^1)$ – vector of pollutants production of power increase of basic production in η_t^1; $D_{22}(\eta_t^2)$ – vector of pollutants production in construction of power increase of purifying devices in η_t^2; $H_1(x_t^1)$ – vector of technological outliers into environment by basic producer in the production process in x_t^1; $H_2(x_t^2)$ – vector of technological outliers of pollutants into environment by purifying devices in case of pollutants utilization in x_t^2; $c^1 > 0$ – vector of natural payment per one worker-producer, $c^2 > 0$ – vector of issue of domestic contaminations per one worker-producer; $f_1(x_t^1)$ – scalar function of economic effect of production in x_t^1; $f_2(x_t^2)$ – scalar function of economic effect of pollutants utilization in x_t^2; $l^1(x_t^1)$ – scalar function of labor resources wastes of products issue x_t^1; $l^2(x_t^2)$ – scalar function of labor resources wastes for utilization of pollutants x_t^2; L_t – total number of workers in period t.

Let us investigate a state of equilibrium for ecological economic system (1). Relevant stationary trajectory of intensity of equilibrium system is determined by increase tempo $\lambda^{-1} > 1$, by Neumann ray $X = (x^1, x^2, \xi^1, \xi^2, \eta^1, \eta^2, y^2, L)$ and obtains the form

$$
\begin{aligned}
&x_t^1(t) = \lambda^{-t} x^1,\ x_t^2(t) = \lambda^{-t} x^2,\ \xi_t^1(t) = \lambda^{-t}\xi^1,\ \xi_t^2(t) = \lambda^{-t}\xi^2,\\
&\eta_t^1(t) = \lambda^{-t}\eta^1,\ \eta_t^2(t) = \lambda^{-t}\eta^2,\ y_t^2(t) = \lambda^{-t} y^2,\ L_t = \lambda^{-t} L.
\end{aligned}
\tag{4}
$$

Let us assume that all non-negative steadily increasing scalar and vector functions acted in model (3), that is $f_1(\cdot)$, $f_2(\cdot)$, $\Phi_{11}(\cdot)$, $\Phi_{12}(\cdot)$, $\Phi_{21}(\cdot)$, $\Phi_{22}(\cdot)$, $D_{11}(\cdot)$, $D_{12}(\cdot)$, $D_{21}(\cdot)$, $D_{22}(\cdot)$, $l^1(\cdot)$, $l^2(\cdot)$, $H_1(\cdot)$, $H_2(\cdot)$ are linear homogenous, thus:

$$f(\lambda x) = \lambda f(x) \text{ when } \lambda > 0 .$$

In this case if we substitute relation (4) into the model (3) for equilibrium position $(\lambda, x^1, x^2, \xi^1, \xi^2, \eta^1, \eta^2, y^2, L)$ with capital T we receive optimal problem

$$\begin{aligned}
&\lambda \to \min,\\
&x^1 \ge \Phi_{11}(x^1) + \Phi_{12}(x^2) + D_{11}(\eta^1) + D_{12}(\eta^2) + Lc^1,\\
&x^2 \ge \Phi_{21}(x^1) + \Phi_{22}(x^2) + D_{21}(\eta^1) + D_{22}(\eta^2) + Lc^2 - y^2,\\
&x^1 \le \lambda\xi^1,\ x^2 \le \lambda\xi^2,\ (1-\lambda)\xi^1 \le \eta^1,\ (1-\lambda)\xi^2 \le \eta^2,\\
&l^1(x^1) + l^2(x^2) \le L,\\
&y^2 \le H_1(x^1) + H_2(x^2),\\
&x^1 \ge 0,\ x^2 \ge 0,\ \xi^1 \ge 0,\ \xi^2 \ge 0,\ \eta^1 \ge 0,\ \eta^2 \ge 0,\ L \ge 0,\ y^2 \ge 0.
\end{aligned} \tag{5}$$

Taking into consideration that $0 < \lambda < 1$ (this special case is the subject of research) we have

$$x^1 \le \lambda\xi^1 \le \frac{\lambda}{1-\lambda}\eta^1,\ x^2 \le \lambda\xi^2 \le \frac{\lambda}{1-\lambda}\eta^2,$$

thus

$$\eta^1 \ge \frac{1-\lambda}{\lambda}x^1,\ \eta^2 \ge \frac{1-\lambda}{\lambda}x^2,$$

and considering the fact that vector-functions $D_{11}(\cdot)$, $D_{12}(\cdot)$, $D_{21}(\cdot)$, $D_{22}(\cdot)$ are non-negative steadily increasing and linear homogeneous, we get

$$\begin{aligned}
D_{11}(\eta^1) \ge \frac{1-\lambda}{\lambda}D_{11}(x^1),\ D_{12}(\eta^2) \ge \frac{1-\lambda}{\lambda}D_{12}(x^2),\\
D_{21}(\eta^1) \ge \frac{1-\lambda}{\lambda}D_{21}(x^1),\ D_{22}(\eta^2) \ge \frac{1-\lambda}{\lambda}D_{22}(x^2).
\end{aligned}$$

Let us introduce the following vector-functions

$$\begin{aligned}
l^1(x^1)c^1 = R_{11}(x^1),\ l^2(x^2)c^1 = R_{12}(x^2),\\
l^1(x^1)c^2 = R_{21}(x^1),\ l^2(x^2)c^2 = R_{22}(x^2),
\end{aligned} \tag{6}$$

that are non-negative steadily increasing and linear homogeneous vector-functions.

It obvious, that

$$x^1 \geq \Phi_{11}(x^1) + \Phi_{12}(x^2) + D_{11}(\eta^1) + D_{12}(\eta^2) + Lc^1 \geq$$
$$\geq \Phi_{11}(x^1) + \frac{1-\lambda}{\lambda} D_{11}(x^1) + \Phi_{12}(x^2) + \frac{1-\lambda}{\lambda} D_{12}(x^2) + (l^1(x^1) + l^2(x^2))c^1 \geq$$
$$\geq (\Phi_{11}(x^1) + R_{11}(x^1) + \frac{1-\lambda}{\lambda} D_{11}(x^1)) + (\Phi_{12}(x^2) + R_{12}(x^2) + \frac{1-\lambda}{\lambda} D_{12}(x^2)),$$

$$x^2 \geq \Phi_{21}(x^1) + \Phi_{22}(x^2) + D_{21}(\eta^1) + D_{22}(\eta^2) + Lc^2 - y^2 \geq$$
$$\geq \Phi_{21}(x^1) + \frac{1-\lambda}{\lambda} D_{21}(x^1) + \Phi_{22}(x^2) + \frac{1-\lambda}{\lambda} D_{22}(x^2) + (l^1(x^1) + l^2(x^2))c^2 - y^2 \geq$$
$$\geq (\Phi_{21}(x^1) + R_{21}(x^1) + \frac{1-\lambda}{\lambda} D_{21}(x^1)) + (\Phi_{22}(x^2) + R_{22}(x^2) + \frac{1-\lambda}{\lambda} D_{22}(x^2)) -$$
$$- H_1(x^1) - H_2(x^2).$$

After multiplication of both parts of the inequalities on $\lambda > 0$ we get the following vector inequality

$$\lambda(\Phi(x) + R(x) - H(x) + (1-\lambda)D(x)) \leq \lambda x, \quad (7)$$

where vector-functions

$$\Phi(x) = \begin{pmatrix} \Phi_{11}(x^1) + \Phi_{12}(x^2) \\ \Phi_{21}(x^1) + \Phi_{22}(x^2) \end{pmatrix}, \quad R(x) = \begin{pmatrix} R_{11}(x^1) + R_{12}(x^2) \\ R_{21}(x^1) + R_{22}(x^2) \end{pmatrix},$$
$$D(x) = \begin{pmatrix} D_{11}(x^1) + D_{12}(x^2) \\ D_{21}(x^1) + D_{22}(x^2) \end{pmatrix}, \quad \mathrm{H}(x) = \begin{pmatrix} 0 \\ H_1(x^1) + H_2(x^2) \end{pmatrix}$$

are non-negative steadily increasing and linear homogeneous ones of vector argument $x = (x^1, x^2)^T$.

Thus, the problem of maximization of tempo increasing balance ecological economic system is considered to be such non-linear optimal model:

$$\lambda \to \min, \quad Q(\lambda, x) \leq \lambda x, x \geq 0, \quad (8)$$

where

$$Q(\lambda, x) = \lambda(\Phi(x) + R(x) - H(x)) + (1-\lambda)D(x). \quad (9)$$

Let us consider in details vector-function $\Phi(x) + R(x) - H(x)$, which is linear homogeneous one. If $H(x)$ indicates vector of sum technological outliers of pollutants into environment by basic production and purifying devices then from economic point of view it is obvious that it is smaller amount of basic production of pollutants of industries, purifying devices and domestic contaminations Then vector-function $\Phi(x)+R(x)-H(x)$ can be non-negative. It our second assumption concerning matrices $\Phi(x)$, $R(x)$ and $H(x)$ (we remind that our first assumption was connected with linear uniform of analyzing scalar and vector functions).

In the work Solow, Samuelson [4] for the first time the non-linear problem about eigenvectors $\lambda v_i = H_i(v_1, v_2,..., v_n)(i = 1,2,..., n)$ is investigated, where all H_i – linear homogeneous functions which are steadily increasing. In Morishima work [5] the following assertion is proved:

Assertion. *Let x – n-dimensional vector $(x_1, x_2,..., x_n)^T$, $H = (H_1, H_2,..., H_n)^T$ is also n-dimensional vector, and its very component H_i is continuous function of x. If every function H_i is non-negative linear homogeneous function of non-negative argument $x \geq 0$, than non-linear problem about eigenvectors*

$$\lambda v_i = H_i(v),\ i = 1,2,..., n$$

has solution in case of non-negative $\lambda, v_1, v_2,..., v_n$, and vector v can be chosen is a way that

$$\sum_{i=1}^{n} v_i = 1.$$

In work [6] we can see such lemma:

Lemma. *Let us have $Q(\lambda) - n\times n$ – matrix, in is definite, continuous on the interval $[\lambda_1, \lambda_2]$, $0 \leq \lambda_1 \leq \lambda_2$. Let γ_1, γ_2 – Frobenius numbers of matrices $Q(\lambda_1), Q(\lambda_2)$ respectively, and $\gamma_1 < \lambda_1, \gamma_2 \leq \lambda_2$. Then the problem*

$$\lambda \to \min,\ Q(\lambda)x \leq \lambda x, x \geq 0,\ \sum_{i=1}^{n} x_i = 1$$

has unique solution $(\bar{\lambda}, \bar{x})$, and:

$$Q(\bar{\lambda})\bar{x} = \bar{\lambda}\bar{x} ;$$

$\bar{\lambda}$ *– Frobenius number of matrix $Q(\bar{\lambda})$;*

$\bar{x}$ *– Frobenius vector of matrix $Q(\bar{\lambda})$;*

$$\bar{x} > 0,\ \lambda_1 \leq \bar{\lambda} \leq \lambda_2 .$$

Using the consideration that lies on the basis of assertion and lemma we can use them concerning non-linear optimal problem (8)-(9). First of all, we put $\lambda_1 = 0$ and $\lambda_2 = 1$. We have $Q(0,x) = D(x)$, $Q(1,x) = \Phi(x) + R(x) - H(x)$. They are Frobenius numbers correspondently $\gamma_1 > 0$, $\gamma_2 < 1$. For vector-function $Q(\lambda, x) = \lambda(\Phi(x) + R(x) - H(x)) + (1-\lambda)D(x)$ all lemma conditions act, is we assume (this is the third assumption about $\Phi(x), R(x), H(x), D(x)$, that vector-function $Q(\lambda, x)$ is productive for all $0 \le \lambda \le 1$. Productivity means mathematically that its Frobenius number is strictly less then one.

Thus, we can affirm that non-linear optimal problem (8)-(9) has unique solution $\bar{\lambda} < 1, \bar{x} \ge 0$.

Let us pass over to search of ecological economic system equilibrium (3).

Let (x, ξ, η, y^2, L) – is arbitrary vector which satisfies inequalities system in (5), which is the solution of the system in condition $\lambda = \bar{\lambda}$. We can demonstrate, that from the conditions $0 < \bar{\lambda} < 1$ and $(x, \xi, \eta, y^2, L) \ne 0$ it is clear, $x \ne 0$. From the system in (5) we have inequalities

$$\frac{1-\bar{\lambda}}{\bar{\lambda}} x \le (1-\bar{\lambda})\xi \le \eta, \tag{10}$$

from where considering $D(\eta) \ge 0$ we get

$$\frac{1-\bar{\lambda}}{\bar{\lambda}} D(x) \le D(\eta). \tag{11}$$

Because $R(x) = l(x)c$, from (5), (11) we have

Because $\bar{\lambda}$ is the Frobenius number, that is clear, that (12) can be true only in the case, when x is the Frobenius vector. It means, that in (12) inequality are transforming into equalities. We get from here

$$R_{11}(x^1) + \frac{1-\bar{\lambda}}{\bar{\lambda}} D_{11}(x^1) + R_{12}(x^2) + \frac{1-\bar{\lambda}}{\bar{\lambda}} D_{12}(x^2) = Lc^1 + D_{11}(\eta^1) + D_{12}(\eta^2),$$

$$R_{21}(x^1) - H_1(x^1) + \frac{1-\bar{\lambda}}{\bar{\lambda}} D_{21}(x^1) + R_{22}(x^2) - H_2(x^2) + \frac{1-\bar{\lambda}}{\bar{\lambda}} D_{22}(x^2) =$$
$$= Lc^2 + D_{21}(\eta^1) + D_{22}(\eta^2) - y^2.$$

Because

$$R_{11}(x^1)+R_{12}(x^2)=l(x)c^1 \le Lc^1,\ R_{21}(x^1)+R_{22}(x^2)=l(x)c^2 \le Lc^2,$$
$$\frac{1-\bar{\lambda}}{\bar{\lambda}}(D_{11}(x^1)+D_{12}(x^2)) \le D_{11}(\eta^1)+D_{12}(\eta^2),$$
$$\frac{1-\bar{\lambda}}{\bar{\lambda}}(D_{21}(x^1)+D_{22}(x^2)) \le D_{21}(\eta^1)+D_{22}(\eta^2),$$
$$H_1(x^1)+H_2(x^2) \ge y^2,$$

the inequalities are true only in the case, when

$$l(x)=L,\ \frac{1-\bar{\lambda}}{\bar{\lambda}}D(x)=D(\eta),\ H_1(x^1)+H_2(x^2)=y^2.$$

Because

$$\eta-\frac{1-\bar{\lambda}}{\bar{\lambda}}x \ge 0,\ D(\eta-\frac{1-\bar{\lambda}}{\bar{\lambda}}x)=0,$$

considering conditions putting into vector function $D(x)$, we get

$$\eta=\frac{1-\bar{\lambda}}{\bar{\lambda}}x. \tag{13}$$

In this case we also have

$$\xi=\bar{\lambda}^{-1}x. \tag{14}$$

It is shown by that way the uniqueness of the Neumann's ray for non-linear model (3), that corresponds increasing tempo $\bar{\lambda}^{-1}$.

Thus, the main trajectory of sustainable development for non-linear expanded model when non-linear model functions are non-negative steadily increasing and linear homogeneous functions is determined in the paper. Also it has been proved the existence of the Frobenius root for non-linear problem according to eigenvalue and according to eigenvector.

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The article was received on November 17, 2008.