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FRACTAL DIMENSION OF INTERNATIONAL FUTURES MARKETS AS A REFLECTION OF THEIR ECONOMIC NATURE

Abstract

The paper analyzes new synergy-based approaches to predicting new pricing tendencies at the international futures markets, and such categories as «attractors», «bifurcation», «deterministic chaos», and «s-fractals». International futures markets are examined as non-linear chaotic systems, the forecasting of their future dynamics with traditional instruments and linear models becoming impossible. In the capacity of an indicator of market tendencies, the author uses the elements of fractal geometry and chaos theory, the fractal dimension in particular.

Key words:

International futures markets, Hurst indicator, price dynamics, traders, fractal dimension, attractors, fractals, market equilibrium, the theory of non-linear dynamic systems.

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It would be better for all of us, if not only in advanced study, but also in everyday political and economic life, as many people as possible realized that simple nonlinear systems not always own simple dynamic properties.

R. M. May

1. The «Crash» of Basic Theoretical and Conceptual Approaches to Market Forecasting in the Foreign Economic Theory of the Late 20th – Early 21st Centuries

Globalization of the world financial markets made their study immensely complicated. Basic economic theories based on rational expectations hypotheses, market demand and supply equilibrium, random fluctuations of market prices, and symmetric information efficiency, turned out to be incapable of making reliable predictions of future price and interest rate dynamics and predicting crisis phenomena, which substantially affect international financial stability and world economic development.

The foreign literature on financial markets analysis ([6], [7], [9], [10], [25], [28]) is based on the linear models that are poorly adapted to trade practice and investment strategies. The application of linear models to basic financial asset valuation and statistical methods of risk determination by means of mathematical expectations and standard deviation explain neither the phenomenon of existence of market trends, nor the time of their origin and change.

The state of foreign economic science at the close of the 20th and the beginning of the 21st centuries was so challenging that scientists and experts started to talk about the failure of the economic theory which had been flourishing in the mid-century. At the turn of the third millennium, a new research trend emerged. It was named the **Non-linear Paradigm,** since its forefathers tried to study financial markets as non-linear chaotic systems in the whole entirety of their complexity and unexpectedness.

The followers of this trend draw on major achievements in mathematics and physics to study such non-linear dynamic processes as *fractal geometry, theory of the determined chaos, cellular automatic machines, neuron networks, fuzzy logic* [2], [15], [18], etc. It was due to these revolutionary achievements in mathematics, physics and computer technologies that the research of sophisticated market phenomena and processes got a chance to be realized on a computer display screen. The works on this issue [16], [26] are still broadly debated by foreign economists, but hitherto disregarded by academic circles. Nevertheless, the market practitioners joined willingly the empiric research of the new hypotheses and received fairly good results [5], [4], [27], [17] [14].

The conventional economic theory turned out to be useless also to the post-socialist economies, including Ukraine with its unresolved problems, such as, in particular, the markets of basic commodity and financial assets which failed to develop throughout the reformation period. We completely support A. M. Romanov [16: 7] in his view that transition from the planned to the market economy encounters a conglomeration of non-linearity. Therefore, the Ukrainian economists and market practitioners need to join the process of national economic theory development through the study of non-linear chaotic systems, which are rather similar to our economy.

Despite this way of setting a problem is quite uncommon, there is no other way to develop national economic theory and form efficient markets. However, choosing this way demands a thorough research of the latest achievements in foreign science, involvement of Ukrainian economists in forming new market hypotheses capable of meeting economic realities. First steps in this direction are made by the Ukrainian economists who conduct research in the field of economic and mathematical modelling. These schools are being developed in Kyiv National Economic University, in Zaporizzhya National University, and Donetsk National University. A Doctoral dissertation «Non-linear Models of Complex Economic Systems» [20] has been defended recently in the latter one.

This new scientific toolkit will give Ukrainian researchers a chance to suggest the measures for formation of starting conditions for subsequent high-quality development of the commodity and financial markets' infrastructure. All participants of this discussion agreed that it would not be possible to form liquid markets without establishing a class of professional traders. However, insufficient are the practical steps taken in this direction. Nevertheless, state authorities gradually change their position in this respect, which can be proved by the fact that the NBU cancelled its veto on speculative currency operations and renewed currency futures trading at the Interbank Currency Exchange. This can become a starting point for the development of a powerful class of financial dealers strongly needed by the markets.

Moreover, the other post-socialist countries, Russia in particular, are actively forming the professional traders of the third wave. The first wave of traders was inherited by the Russian financial market from the USSR. These were the currency and «gold» dealers of Zovnishekonombank, who carried out the entire spectrum of operations with gold and currency at the international exchange markets. The professionals of the second wave learned to work at the international spot currency market FOREX. The third wave of financial dealers succeeds not only at the FOREX, but also at international and national stock, commodity and financial futures and options markets. Their professionalism is reflected in the studies they published [3], [11–13], [31], [30].

Because of the lack of advanced research in forecasting of prices and rates, these traders, apart from successful trading practice, carry out studies impressive in their novelty. Good evidence is a preface to a two-volume book «Financial Dealing», by V. Yakymkin [30]. The preface was written by O. Elder, the avowed authority within the international circles of exchange professionals, the former political emigrant from the USSR, and the author of the bestseller «Trading for a Living», which ran through 8 editions including the Russian-language edition under the title «How to Play and Succeed at the Exchange» [29]. In the above-mentioned preface, O. Elder appreciates the Russian traders' work at high value and underlines the fact that the professionalism of Russian dealers is growing so that after a short time their books will be translated into English.

In our opinion, the time has come in Ukraine to transform completely the approaches to establishing the class of financial dealers. Today, they do not have the possibility of carrying out operations at the global financial market and, as a result, lose in a professional sense to their competitors, since without knowledge of modern techniques of managing financial innovations, they do not get additional profits and cannot master effective financial risk management methods.

To meet these challenges, a comprehensive approach should be taken, in particular: to research thoroughly modern international financial markets, especially their term segments; to make proper amendments to legislation, and to carry out measures on training and retraining financial personnel.

More and more of the modern studies compare international financial markets with living organisms because of their sophisticated functioning and structure. Forecasting of future prices and rates is as difficult as is it to make a precise weather forecast. ***The only irrefutable fact is market uncertainty.***

In this paper, the author attempts to analyze the existing approaches to futures prices forecasting and to show how the elements of non-linear economy, namely fractal geometry and the chaos theory, could be used in this process. In order to do this, the author calculates the fractal dimension of some foreign futures markets, as well as commodity and financial futures.

2. Theoretical and Conceptual Approaches to Predicting Futures Prices Forecasting

The critical analysis conducted by the author shows that the basic hypotheses describing the formation and functioning of both the futures markets and the term segments of the financial market are the ***theories of rational expectations of market participants, market equilibrium, market information efficiency, and random price walk,*** that is, exactly those criticized by the authors of non-linear paradigm.

Let us mention briefly the basic postulates of these theories (a detailed analysis of the major of them was published in «Economika Ukrayiny» [23] and «Bankivska Sprava» [22]).

The hypothesis of rational behaviour of the person participating in an economic process was suggested by J. M. Keynes [34] and J. R. Hicks [33], though even at that time the former of them did not consider it correct. The main behavioural motive of a person in the economy is his/her expectations. The future state of the economy can be forecasted by means of *adaptive (retrospective) and perspective expectations.* The theory of adaptive expectations assumes that the future of an economy with its attributes (prices, interest rates, inflation rates, etc.) is developed exceptionally on the basis of *past information analysis.* This hypothesis generated a specific direction of market analysis – **the technical analysis** of market prices and volumes of trade with its basic postulate: *«Market is aware of everything, keeps everything in memory, and submits to trends – tendencies».* At present, technical analysis is a primary instrument for one-day traders- speculators at the global spot market FOREX and at international futures and options exchange markets. Position speculators and hedgers use it when determining the time of opening and closing positions.

The theory of perspective (rational expectations) assumes that a market participant uses all accessible information – be it *past or current or insider.* This theory, regardless of J. M. Keynes' criticism, laid the foundation for **fundamental analysis** and gave rise to *the theory of market information efficiency,* the «fashion» of the last century's 80-s. Fundamental analysis aims at determining an asset's «fair price», which, after being compared to the market price, allows defining whether the valuation is accurate and taking a proper purchase/sell decision.

Most of the modern economics textbooks include a chapter on the ***theory of market equilibrium,*** according to which a market is moving toward demand and supply equilibrium, which is expressed in the equilibrium price [7]. This theory does not eliminate deviations from the equilibrium state. Practically, the market is constantly affected by discontinuous information and, as a result, it changes its form. Moreover, information is also asymmetric, imperfect for market participants. Foreign economists view this fact as the reason for market disequilibrium. The importance of these works is proved by the fact that J. Stiglitz was rewarded Nobel Prize in 2001 for his research «Equilibrium at Competitive Insurance Markets: Essay on the Economy of Imperfect Information» [24].

In reality, market equilibrium is more likely a random than permanent phenomenon, since demand and supply of the majority of primary products and financial instruments are inelastic in terms of price changes. One can repeatedly observe that market trade volumes increase when prices increase and vice versa. One reason to it is that not only assets, but also expectations of their future prices and rates, are traded at the markets.

Uncommon is the view of the trader and analyst E. L. Niyman [12: 105], who uses the philosophical perception of equilibrium (i. e. the absence of motion means death) to characterize non-liquid equilibrium markets with low volatility as «dead markets», as, for example, was the Ukrainian stock market in 2001. However, this market was «alive» even then, since the volumes of supply and demand fluctuated, though hardly noticeably. The imperceptibility of these changes was explained by the fact that only about 8% of operations were conducted through formal infrastructure of the stock market [1]. That is why *the theory of dynamic equilibrium as* ***a market objective*** *has all rights to exist.* Similar to nature, which accumulates its strength before evolutionary changes, the markets enter the phase of consolidation after significant fluctuations.

In 1972, E. Fama [32] suggested the hypothesis of **market information efficiency,** which stated that as long as all new information is equally accessible to all market participants it is already accounted for in the price of an asset. This hypothesis, in spite of being used for quite a long time at the US stock market, failed the practice test, particularly because it disregarded the psychology of market participants and their unequal perception of information. One of the most rigid opponents to this hypothesis is G. Soros, who asserts that «when we apply economic theories to the real world, we can get a distorted picture. This is especially noticeable in the financial markets, where reflectivity plays an important role. The theory of rational expectations and market efficiency is deeply incorrect». [21: 237].

Being the maker of the *theory of market reflectivity,* which is based on the psychology of market participants, he criticizes economists who recklessly apply the laws of physics to the market. He particularly disagrees with the statement that perfect knowledge provides perfect information, as well as ignores the theory of random price walks.

The theory of random walk proves that market prices vary randomly since they are formed under the influence of unexpected information. This hypothesis was empirically tested at the US stock market, revealing the following regularity: the linear charts of closing prices quoted on stock exchanges turned out to be similar to the charts of the Brownian motion, which allowed asserting that market prices are absolutely random, and it is impossible to forecast them. This hypothesis presumed wide application of game and probability theories.

Mathematical approaches to estimation of random values through standard deviation and mathematical expectations started a new direction in financial theory called the *theory of investment portfolio.* H. Markovitz became the author of this theory [36]. In 1952 he developed a *probabilistic model* of stock market with basic notions of *profit* and *risk.* The profit derived from stock portfolio was determined by means of mathematical expectation, while the risk of failure – through standard deviation of expected profit (variance). The most valuable feature of his approach was the dependence of an optimal investment portfolio composition upon *various initial conditions*, which however did not correspond closely to the random walk hypothesis. In forty years the mathematician H. Markovitz was awarded Nobel Prize in Economics for this work.

J. Tobin developed H. Markovitz' ideas by introducing the notion of a risk-free asset (risk-free loans and risk-free crediting), which provided an opportunity to reduce investment risks by allocating funds in risky and risk-free assets. J. Tobin was also awarded Nobel Prize for his research even earlier than H. Markovitz.

The other Nobel laureate W. Sharp [28] suggested *a one-factor linear model of capital market,* known to economists as the CAPM (Capital Asset Pricing Model – *the model of capital asset valuation),* which introduced α and β coefficients for description of return on certain stock. The model sets a correlation of risk and return at the near-equilibrium market. The CAPM is used to determine the systematic (market) risk and the riskiness of a certain share, which could be minimized by portfolio diversification. Graphically, the model is represented by a straight line that illustrates the direct dependence of return upon risk. As can be seen, V. Sharp suggested a linear model of the market, which is based on the market equilibrium hypothesis, the probability theory, and the efficient market hypothesis.

S. Ross made the latter hypothesis a basis for the Arbitrage Pricing Model (APM). The model rests on the assumption that at every point of time there is such a risk/return correlation on the market that the participants can not gain a risk-free profit from arbitrage (deals on price variations). As we see, this model also grounds on the random walk hypothesis and states that price variations are chaotic, and the accurate forecast of profit is impossible [25], [28].

Although the above-mentioned hypotheses are studied at higher educational establishments of the USA and Europe and applied by institutional investors and stock exchange traders, they are not confirmed in practice, which urge scientists and market participants to search for new ways of solving these complicated problems.

In testing the random walk hypothesis, according to which price fluctuations at financial markets are similar to the Brownian motion, B. Mandelbrot [35] has already in 1964 proved, on the basis of US stock market returns analysis, that these returns are distributed in conformity with the Pareto's, not Gaussian, law, as they are characterized by indefinite or infinite dispersion. However, most of the economists found his research incorrect and the deviation from the Gaussian curve unimportant. As it is known, deviation from the mean is normally distributed, which is graphically represented as the bell curve, named after its author the Gaussian curve.

This was the case for another decade, until E. Fama (1965) and W. Sharp (1970) noticed that the distribution of stock market returns had negative asymmetry: «tails» were thicker, while the spike around the mean exceeded the value predicted by normal distribution. Having investigated more thoroughly the US stock market volatility of daily S&P and Dow Jones indices for the period from January 1928 till January 1990, A. Turner and E. Wegel arrived at similar conclusions [16:47].

Thus, at the early 90s of the 20th century, a group of economists developed a contrary to conventional viewpoint on the fact that standard deviation is not a measure of financial risk primarily beyond short-term periods of time, while the increased volatility (price variations) of multi-investor markets denies the theories of rational expectations and market information efficiency.

3. Synergetic Approach to International Market Research

As it was mentioned above, the late 20th century was marked by revolutionary discoveries in physics and geometry, allowing better understanding of the phenomena of nature. What is meant here is the development of a fundamentally new scientific paradigm, which asserts that order originates from chaos, fluctuations are the sources of organization, and uncertainty plays a constructive role in evolution. Diversity is the foundation of steady and dynamic development of systems. Evolution is non-linear and multi-choice. The picture of the world of conventional science – the Laplacian determinism – appears as a «parody of evolution» to I. Prigogine [19], the maker of the synergetic approach. The pattern of scientific thought changes as well. Synergy purports to reveal the universal mechanisms of self-organization, which makes it a cross-disciplinary line of research. It claims the world is extremely complicated, open, and unstable, since it is continuously emerging. All linear and organized is a mere exception, while non-linear and chaotic is a rule of existence for both the nature and the society.

Synergy introduces its own language into the scientific practice. It is the language of such categories as ***attractors, bifurcations, determined chaos, and fractals.***

The theory of chaos, quantum physics and fractal geometry proved the fallacy of determinism as a symbol of scientific perception of the world true only in certain situations. On the contrary to the viewpoint which perceived probability as incomplete knowledge came the understanding of it as an objective reality.

The ***theory of chaos or the theory of non-linear dynamic systems*** is attributed to E. Lorenz. In studying the processes of exact weather forecasting, he proved it impossible to predict future because of unavoidable measuring errors, which in their turn result from incomplete knowledge about initial conditions. ***Chaos*** is defined by Lorenz as *the extreme uncertainty of permanent non-linear irregular complex motion.* According to the chaos theory, randomness and order coexist in the world. The unpredictability of chaos is explained with its intense dependence upon initial conditions. At the same time, chaos is not random.

The study of non-linear dynamic systems and the *complexity theory,* with the theory of chaos and fractal geometry as its constituents, implies the research of the process of a system's transition from the stable state to turbulence, which is impossible to explain by the Newtonian physics. The latter can predict the orbits of planets for centuries ahead, but it appears powerless in making a weather forecast for several days to follow [8].

Having applied this postulate to financial markets, we can assert that price dynamics is chaotic. It is not random, but nevertheless, it is unpredictable. Small changes and (or) errors in defining initial conditions can produce great effects when the uncertainties accrue and exceed the forecasting limits. In the theory of chaos, this phenomenon is known as **substantial dependence on initial conditions** or *exponential error accumulation.*

Another important thing is that the reliability of forecast falls quickly over time. The **concept of critical levels** is no less important as well. This idea represents a non-linear influence of some factors upon the system causing its collapse. To illustrate this phenomenon, E. Peters [16: 162] gives a classical example of a straw which broke a camel's back in two parts. The straw's weight, added to the great weight carried by the camel, exceeded the critical level of animal's abilities and killed it.

The theory of chaos studies the phase space which is the abstract space, the co-ordinates of which are the degrees of freedom of a system. To determine the degree of the chaotic system's freedom, E. Lorenz used such instruments as ***attractors.*** Attractors are the geometrical structures which originate from the system's behaviour in the phase space over a long period of time. *Attractor is the range of a system, the limit of its fluctuations and dynamics.* In other words, the attractor is the system's decision region. The simplest attractors are ***a point, a circle, and a torus***. These belong to simple foreseeable attractors.

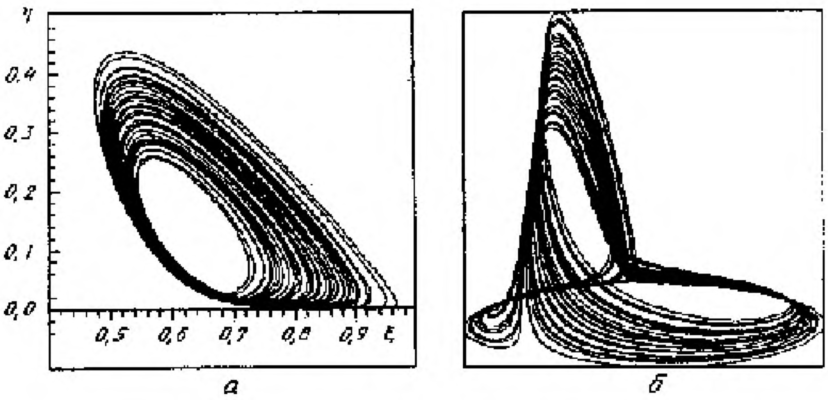
However, the time-series in nature and economy are characterized by non-recurrent cycles; therefore, they are complex dynamic systems with infinite solution options. Their behaviour in the phase space is described by means of chaotic attractors.

E. Lorenz constructed a first chaotic attractor by solving three differential equations with three constants and different initial conditions. Proceeding from calculations and graphing the results on a three-dimensional system of coordinates, it was illustrated that the solutions of these equations are chaotic, since a slight difference in initial conditions resulted in substantial divergence of the received results.

Figure 1 shows **chaotic attractors** [8]. As the Russian scientists of Keldysh Science Centre assert, one can just wonder at E. Lorentz' scientific courage. For weather forecasting he used a simple model of three differential equations, which, after a subsequent computerized calculation, gave him a clear vision that he dealt not with calculation errors, but with a scientific discovery. The mathematical shape of determined permanent processes, which are impossible to predict over the long-run, was referred to as a «strange» attractor.

Figure1

A Two-Plane Projection of Chaotic Attractor Describing Object's State in a Three-Dimensional Space [39]



Convergence and divergence (compression and extension) of a chaotic attractor systematically substitutes the initial information with a new one. When the attractor's trajectories converge, the uncertainty of information that is large in volume increases, and vice versa, when they diverge, grows the uncertainty of the small-volume information. Proceeding from the above, we can get the following conclusion: the chaos theory denies causality between events, which eliminates the possibility of predicting the system's future behaviour. As a result arises yet another conclusion: the longer is the time interval under forecast, the greater is the probability that the forecast is wrong.

To analyze financial markets, E. Peters used chaotic or *erratic (chimerical) attractors*. There are several Ukrainian-language web-sites in the Internet that provide books (Anisimov I. O. «Vibrations and Waves») and seminar materials dedicated to fractal study in Cherkasy State B. Khmelnytskyi University, as well as dissertations on the problematic of chaos and fractals in mathematics and physics [39]. These materials prove a considerable interest of Internet-users in these issues.

Chaotic attractors generate such behavioural patterns of the system which can neither be predicted nor reproduced. Any phase space occupied by erratic attractor is characterized by complex behaviour in time. Therefore, to determine these chaotic systems, the notion *of time horizon* is introduced. It exposes principle differences between the past and the final states of a system. Consequently, we can assert that the chaos theory specifies the notions of predictability and probability in the financial market model.

This theory, in our opinion, is the most promising line of international financial markets research. However, as it was already mentioned above, not all western researchers and practicing traders unambiguously accept its methods. The Ukrainian analyst E. Niyman [13: 272] supports them by stating that as long as there are no mathematical instruments for chaos theory application in market prices analysis, there is no need in rushing for its application. However, in 2004, he wrote a book under the eloquent title «Trade Chaos», which is yet to appear on the book market.

Another very significant fact is that successful foreign traders, who use advanced ideas for market competition, write about it rarely. B. Williams is an exception [4, 5].

The numerical measure of chaos is the velocity of convergence and divergence in the system, while the statistical measure is the dimension of an attractor. It is impossible to define the statistical dimension of chaotic attractors by means of the Euclidian geometry, since they are dynamic structures characterized by unsteady and non-periodic shifts in trajectories. With that end in view, the chaos theory applies **fractal geometry** that permits to analyze indistinct and incorrect forms. The founder of this new trend in mathematics was B. Mandelbrot [39]. He was the first to introduce the term ***fractal***, having combined two Latin words: *«frangere»* – to break, *«fractuc»* – to create irregular fragments. He ascribed this term to a geometrical structure which would allow describing more precisely the various objects of the nature which is similar, relative, and chaotic at the same time.

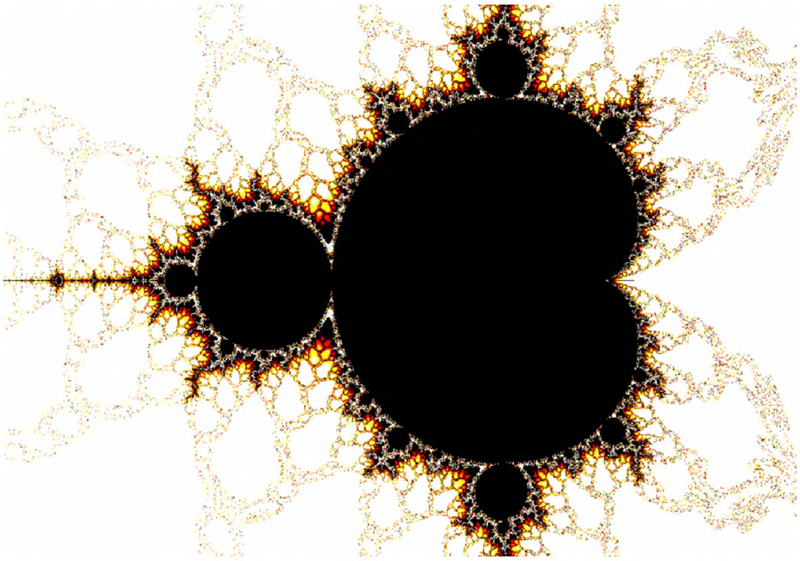
When studying different natural objects, we often observe a statistical self-similarity of mountains, rivers, trees, flowers, etc. B. Mandelbrot proved that the shape of England's coastline can be described in a manner that conforms to the law of degree, while the Brownian motion resembles an infinite, undifferentiated curve. Figure 2 presents the Mandelbrot fractal. One can hardly believe that this lovely picture was generated by a computer solving a simple equation $x_{n+1} = x^2_n + c$, where c is a complex number. This is a flat fractal, which has a fractional dimension.

The Mandelbrot's discovery of the universal figure *M* permitted to demonstrate the immense complexity contained in a simple law. Every point of the figure is marked with self-similarity; the set is continuously reproducing itself in different sizes, but always in the one and only form.

The Mandelbrot's revolutionary invention allows the scientists of different fields of knowledge, who study physics of plasma, metamorphosis of plant development, psychology of individual's perception of various events, social behaviour, etc., to ascertain that the structure formation processes and self-organization follow in their development a small number of scenarios, which do not depend on a specific system. The study of non-linear dynamic processes brings us to the study of complex natural phenomena. The words of R. M. May cited in the epigraph to this article are pressing even today. In 1976 he said: «It would be better for all of us, if not only in advanced research, but also in everyday political and economic life, as many people as possible realized that simple non-linear systems do not always own simple dynamic properties» [16].

Figure 2.

Mandelbrot Fractal



All objects in the nature are rather complicated. Their complexity grows as we approach them. Thus, a leaf and a flower of any plant will only resemble other plants of this kind, but they will always have their unique forms and sizes. At that, the closer we examine them (with microscope for example) or the further we move away from them, we will either observe essential distinctions or simplify and perceive them identical. This is the way the Euclidean geometry does it. A three-dimensional form transforms into a two-dimensional one with subsequent transformation into a one-dimensional line or a dimensionless point. Fractal geometry lets us see not only similarity, but also complexity of objects at their detailed study.

When the Mandelbrot set is constructed given that the complex number *C* has different values, the limit between the two zones of infection takes on a fractional structure, where a single form has different sizes and is observed in different places. These limits in mathematics are said to be the Gaston Julia's sets.

During the times of the World War I, G. Julia proved that the entire limit could be reproduced from its tiniest part by using the final number of iterations of $x \to x^2 + c$. Another feature is that these sets encompass extremely complex dynamics. The students of Cherkasy State University [39] created a gallery of fractals, both flat and spatial. Figure 3 shows one of the spatial fractals of Julia's sets, illustrating how complicated the spatial attractor of the system is, although in terms of mathematics, the process is described by the same equation.

Figure 3.

Spatial Fractal of Julia Set [39]

The first who made an attempt to apply the accomplishments of fractal geometry to the analysis of financial markets was E. Peters [16], [37], whose works have already been mentioned. It was him who started the discussion about the paradigm shift in the study of financial theory.

Taking into account such two basic properties of fractals as *self-similarity and divisibility,* he started to analyze the charts of price changes over time, that is, the technical analysis charts. Self-similarity in all segments of the financial market can be observed when comparing price diagrams of different assets or when one asset is compared in different time intervals. The property of financial markets to exist simultaneously in several time dimensions allowed E. Peters to put forward **the fractal hypothesis** [37]. The fractal nature of the financial market makes it possible to represent it in the form of *spatial and temporal fractals.* The charts of price changes over time are used as spatial fractals. As temporal fractals used are *the investment horizons (asset holding periods)* of major market participants.

Trade strategies of the speculator and the investor differ in the length of investment horizon. One-day speculators and arbitrageurs at the FOREX markets and futures exchanges open and close their positions during an auction day for a short period of time, i. e. for a few minutes or a few hours. Position speculators hold their positions from a couple of days to weeks and months. Stock and futures markets investors and hedgers are present at these markets from few months to few years. Strategic investors can have an unlimited investment horizon.

Thus, different investments horizons (temporal fractals) of the market participants allow increasing market liquidity. Only the presence of a great number of financial market participants that pursue their own investment strategies explains the fact that assets are simultaneously bought and sold. If strategic investors or hedgers leave the market, the position speculators (particularly day speculators) will have no counteragent to conclude agreements with, since all of them use the same graphs of technical analysis (spatial fractals) with the same time interval for setting opening, maximal, minimal, and closing prices, for example, a 5-minute chart. Having received a buy signal (expected price rise), all of them will try to purchase an asset but they will have no one to sell it to them. The same degree of illiquidity belongs to the Ukrainian foreign exchange and stock markets, where there are almost no speculators dealing within short investment horizons.

Fractality, which is characterized by general certainty and local randomness, belongs not only to natural objects; it is the basis of their existence and resistibility to internal and external influence. ***By analogy with nature, the resistibility and liquidity of financial market is determined by the variety of investment horizons of its participants. The real financial market is not determined by the law, and at the same time, it is not utterly random. Its measure of uncertainty is its fractal dimension.***

Fractal dimension shows how the time series fills the space. The way how the object fills in the space depends on the forces which determine its formation. To calculate fractal dimensions, the object must be multiplied by n times to find the number of its copies. A straight line has dimension of 1. This proceeds from the fact that a segment of line can be multiplied by X and then divided into Y copies of the initial line; at that, $Y = X$ or log 2 / log 2 = 1.

The fractal dimension of flat objects is determined similarly. For example, let us take a square and multiply its side by 4. The acquired square contains 16 copies of the original size square, that is, the enlargement of linear dimensions of a flat figure by x multiplies its area by x^2, or log4 / log2 = 2. Similarly, volumetric figures have fractal dimension of 3 = log 8/ log 2.

The general fractal dimension formula is the following:

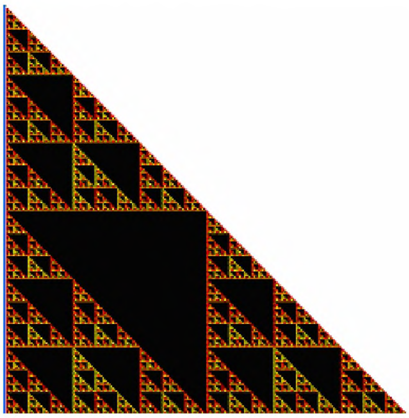
$$\log n / \log 1 / r,$$

where n = the number of self-similar parts that appear when the figure increases by r.

Fractal forms can be created in different ways. The simplest illustrations of these fractals are represented by the Serpinski triangle and the Kokh snowflake. Figure 4 shows Serpinski triangle constructed by the students of Cherkasy B. Khmelnytskyi State University [39].

Figure 4.

Sierpinski Triangle



The Serpinski Triangle can be obtained by successive excision of equilateral triangles from the basic one, as it is shown on the figure. The Kokh snowflake is constructed within a circle, where equilateral triangles are consistently inscribed on every middle third of the side of the previous one. The contour's length is infinite, although the flake is placed within a circle of definite size. The illustrated figure is called symmetric fractal. However, natural objects are never symmetric. Natural objects are **random fractals.**

The fractal dimension of such configurations as, for example, a coastline – the contour of which at certain cruising altitude transforms from a continuous curve into a broken line with a lot of notches – is determined by measuring its notches. This contour resembles graphs of many processes (in particular, price charts of financial markets); therefore, it is important to determine its dimension. It will be more than 1, but less than 2. This dimension is determined by counting the number of circles necessary to cover the coastline. The amount of circles is determined as:

$$N\,(2r)^D = 1,$$

where N = number of circles;

r = radius of circle;

D = fractal dimension.

Consequently,

$$D = \log N / \log 1 / 2r.$$

The fractal dimension of the Koch snowflake is 1.26, of British coastline – 1.3, of Norway – 1.52, which testifies that the latter has more notches [16: 82].

B. Mandelbrot calculated fractal dimension of motion of the Brownian particle. Mathematical expectation of the Brownian motion is proportional to square root of the time of increase. Dividing the time interval in n equal parts, and vertical axis – in subintervals Δt, he determined fractal dimension according to the following formula:

$$D = -\lim (\log N\,(\Delta t) / \log \Delta t = 1{,}5, \text{ at } \Delta t - 0.$$

The Russian currency trader V. Yakymkin [29: 30], having studied the fundamentals of fractal geometry and behaviour of chimerical attractors, calculated the fractal dimensions of the global currency market FOREX on different time plots.

He started his research with verifying the probabilistic model of the financial market and thoroughly studied the investment horizons of traders as real objects of fine structure. This structure turned out different for the same asset. Thus, the buyers dominate on one time allotment (10 minutes), while within another (one hour) dominate the sellers. Despite investment horizons are correlated, it is impossible to predict price movements over a long-time apportionment on the basis of a short–time one.

Having applied the portfolio approach to currency trading, he suggested opening several sell and buy positions, proceeding from probabilistic estimation of future market conditions: increase of rates, uncertainty, or drop (long, flat, short), which makes possible a considerable reduction of risk without decreased return. As we see, this researcher continues to use the probabilistic model of financial market as the basic one.

Then he tried to test the hypothesis of fractal market through the prism of price movement and simulation of investor behaviour by calculating the fractal dimension for USD/CHF daily price graph (daily closing prices for the period from January 1999 till June 2000). In computations he used the Hurst index [16: 99], which shows the maximal range of price change over certain time interval (H that changes within the limits of $0 \leq H \geq 1$). Hurst discovered that the value of H always exceeds 0.5, and the distribution of successive values is abnormal. At $0.5 \leq H \geq 1$, the time series is characterized by stable trends, implying that such series have long-term memory. In other words, the place where we are now is determined by the place we were in the past.

Hurst index can be transformed into a fractal dimension through formula:

$$D = 2 - H.$$

This formula means that fractal dimension of a flat series is always less than 2 (dimension of a plane), and more than 1 (dimension of a curve). According to Mandelbrot's calculations, the fractal dimension of the time series of random price walk makes $D = 1.5$. Therefore, at $1 \leq H \geq 0.5$, the geometrical interpretation of the series will approximate a relatively smooth curve with small notches. At $0 \leq H \geq 0.5$, the series will have high value of fractal dimension, that is, it will have even more notches than does the random walk series.

V. Yakymkin's [30: 50] calculations showed that for a given USD/CHF price series, H equals 0.5036 and fractal dimension 1.4964, accordingly. This made the author come to a rather incorrect conclusion that the price series of the two currencies conforms to Gaussian distribution.

V. Yakymkin examines the fractality of FOREX through the self-similarity of the curves of normal distribution of average statistical price dependence upon volatility, irrespective of time horizons. Therefore, his approach substantially differs from that of E. Peters, who considers that the statistical determination of risk as the standard deviation of income or price volatility needs considerable correction [10: 84]. Random price changes imply that people immediately (linearly) respond to information, which is mostly not true in practice.

At the FOREX market and the futures segment of the financial market, investors perceive the same information differently, depending on the length of their own investment horizons. Thus, daily price or rate changes will be viewed as insignificant by a hedger, a position speculator, or a strategic investor, but they will call for immediate reaction of a day trader, which can lose a lot or even be pushed out of the market because of negative dynamics.

Investor behaviour at stock markets is distorted by insider information. Having insider information, investors react to news in a non-linear way. Moreover, as it was already mentioned above, the so called asymmetrical information exists for sellers and buyers on all segments of the financial market, which also causes specific reaction.

E. Peters clearly proved that the price charts for financial assets are not similar to those of random variations. As a rule, their series are skewed, and the scope of deviations from the mean exceeds the Gaussian normal distribution.

V. Yakymkin's calculations of different price series for different time intervals and his constructions of normalized bar charts for the pair of currencies, S&P 500 closing prices, and US state bonds proved that the distribution curves have higher and skewed to the right spikes and thicker «tails» than does the Gaussian curve.

4. The Applied Aspects of International Futures Markets Research

Figures 5 and 6 show the futures market price charts for financial (Dow Jones index futures contract with delivery in March 2003) and commodity assets (corn futures contract with delivery in March 2003) at one point of time (26.12.2002) within several temporal dimensions (10-minute, daily, and weekly price sets) at the Chicago Chamber of Commerce (CBOT) [38]. The form of the charts, their general self-similarity and local difference prove their fractality, that is, these charts can be considered as flat fractals.

Thus, Figure 5 demonstrates fractal silhouettes on December 26, 2002, which will interest the traders of different investment horizons. The charts clearly illustrate that at one point of time the dynamics of price movements and the trends, or market profile, will be different. The upper chart, which represents the three-year (2000–2002) price dynamics (weekly opening, closing, maximal, and minimal prices) for the Dow Jones Industrial Average futures contract with delivery in March 2003, shows that the drop of the predicted 2002 index values in August changed to growth which continued until mid-December. This chart was generally used by fundamental analysts of the US stock market, strategic investors, and corporate managers to predict the state of the economy in March 2003.

The middle chart illustrates the six-month price change dynamics for the same futures contract, but with daily price fixation. This chart demonstrates the beginning of the rising trend. It is used by portfolio investors, position speculators, and investment managers of mutual funds for hedging the portfolios composed of the index stock.

The lower chart illustrates price dynamics of the same index for two closing days with 10-minute price fixation. This chart is used by day speculators (day traders and intraday traders) holding sell/buy positions for several minutes to several hours. As seen on the chart, on 23.12.2002 at 9 a. m. a trader could open a seller position for several contracts at the price of 8540 points and redeem it (i.e. close the position by reverse agreement) at 8460, getting 100 pips of gain, which corresponds to $10000.

Figure 5.

Fractal Silhouettes of the Futures Market for Corn

Figure 6.

Fractal Silhouettes of the Futures Market for the Dow Jones Index

In having earned a speculative profit, the trader performed an important economic role. He acted as a counterpart to the investor with longer investment horizon and, thus, helped to maintain market liquidity. When the investors with longer horizons get signals of price rise, all of them will buy and no-one will sell. In this case, the market can seize to exist. Consequently, the market survival and its liquidity depend on the presence of sellers and buyers with different asset holding periods.

Figure 6 shows the charts of price changes for the corn March 2003 futures contract. A 10-minute chart can become the item of interest for day speculators and arbitragers, who get profits or losses from their long-term or short-term positions held for several minutes, hours, or days. Thus, as we see from the chart, on 23.12.2002, at 9.30 a.m. one could open the buyer position for 5 contracts (it is sufficient to make an initial margin payment of $1200) at the price of 239 cents per one bushel (a contract covers 5000 bushels – 127 tons) and at 13.30 p. m. he could close it by the reverse agreement at 243 cents per bushel, earning 4 cents of gain on a bushel, which makes $1000 for 5 contracts. As we see, the profitability rate is rather high, since the stockbroker earned $1000 within four hours from $1200 of initial investment. However, had he opened the seller position, that is, would his expectations of price changes have been the opposite, he could have lost a similar amount of money, having only $200 remaining on his account.

A weekly chart of price dynamics (the upper chart) will keep major corn market participants (producers, traders, processors, and government) interested. The three-year graph (2000-2002) demonstrates clear trends of price ups-and-downs related to harvesting seasons, sowing campaign, etc. At that, during these three years, the price for corn with March delivery fluctuated within the range of 180–280 cents per bushel, which proves high historic market volatility and risk, which, in its turn, made these futures popular for hedging off immense price risks.

The middle graph is used by corn traders who insure corn stock from depreciation, which just had to be done under the price fall from 300 to 240 cents per bushel in August-December 2002. As we see, Ukrainian agrarians are not the only ones who suffer from seasonal fluctuations at the corn market, but their American colleagues have a safety mechanism. Therefore, several months before harvesting, they open seller positions at high prices and close them after having realized their corn at the real market at low prices. As a result, they generate profit at the exchange and compensate losses from the sale of the cheapened corn.

Hence, all the abovementioned futures market charts for different assets on different time plots are self-similar and randomly individual at the same time. These markets exist in several time dimensions at the same time, and this proves that the futures segments of financial markets are fractal objects.

On the basis of market volatility (price fluctuation) of the Dow Jones Industrial Average futures contract for January 1988 – July 2003 and the futures market for corn for the January 1980 – July 2003 period at Chicago Chamber of Commerce [38], we will calculate the fractal dimensions and Hurst indices for monthly series. We will show the results of computations in Tables 1–4 and Figures 7 and 8.

Figure 7.

Volatility of Dow Jones Index Futures Market (1988–2003)

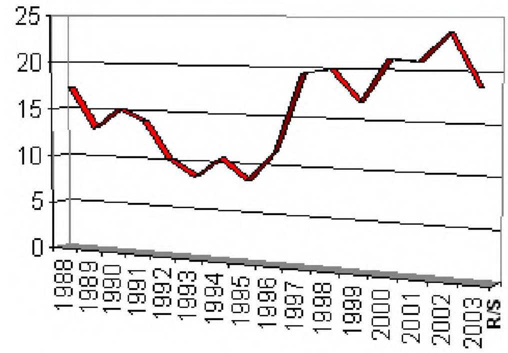
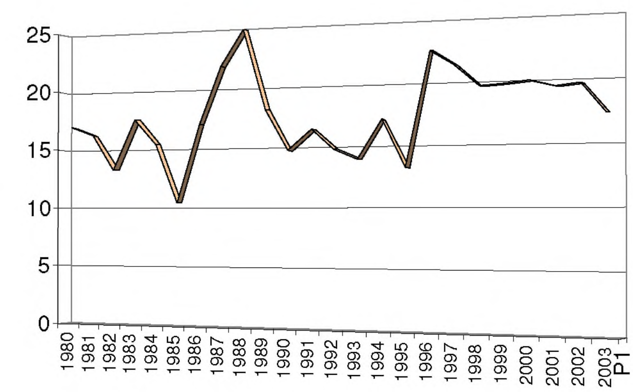


Figure 8.

Volatility of Corn Futures Market (1980–2003)



Proceeding from the computational data we see that the Hurst index for futures market for corn (this index measures the influence of information upon the time series data) prior to 1996 fluctuated within the 0.49–0.61 range, and after 1996 it changed within the 0.66–0.61 range, which testifies to the presence of yearly persistent series characterized by persistent trends.

Table 1.

Computation of Fractal Dimensions of Dow Jones Index Futures Market in 1980–2003

Years	*R/S*	*H* = lg(*R/S*) / lg(365/2)	*D* = 2 – *H*
1980	16.9	0.5430093	1.4569907
1981	16.2	0.5348847	1.4651153
1982	13.2	0.4955523	1.5044477
1983	17.5	0.5497097	1.4502903
1984	15.4	0.5251582	1.4748418
1985	10.4	0.4497634	1.5502366
1986	17	0.5441424	1.4558576
1987	21.8	0.5919066	1.4080934
1988	24.8	0.6166694	1.3833306
1989	18.1	0.5561842	1.4438158
1990	14.7	0.5162236	1.4837764
1991	16.4	0.5372413	1.4627587
1992	14.8	0.5175257	1.4824743
1993	13.9	0.5054763	1.4945237
1994	17.1	0.5452688	1.4547312
1995	13.2	0.5532863	1.4467137
1996	22.5	0.6676436	1.3323564
1997	21.3	0.6558909	1.3441091
1998	19.6	0.6380548	1.3619452
1999	19.7	0.639146	1.360854
2000	19.9	0.6413121	1.3586879
2001	19.4	0.6358554	1.3641446
2002	19.6	0.6380548	1.3619452
2003	17.3	0.6112885	1.3887115
Value			
Average	17.52916667	0.5363751	1.4636249
For 23 years (1988–2003)	17.5	0.5497097	1.4502903
Variation	0.029166667	–0.0133345	0.0133345
Max	24.8	0.6166694	1.5502366
Min	10.4	0.4497634	1.3833306

Table 2.

Computation of Fractal Dimensions and Hurst Indices for Annual Series of Price Volatility of the Dow Jones Industrial Futures Contract in January 1980 – July 2003 (Monthly Volatilities)

	January (1988–2003)	February (1988–2003)	March (1988–2003)	April (1988–2003)	May (1988–2003)
R/S	17.2	14.7	16.4	16.2	13.9
H	0.417375423	0.394332941	0.410387932	0.408587792	0.386123271
D	1.582624577	1.605667059	1.589612068	1.591412208	1.613876729

	June (1988–2003)	Jyle (1988–2003)	August (1988–2002)	September (1988–2002)	October (1988–2002)
R/S	13.1	14.6	16	16.1	19
H	0.377426825	0.393331507	0.420532382	0.421477401	0.446597769
D	1.622573175	1.606668493	1.579467618	1.578522599	1.553402231

	November (1988–2002)	December (1988–2002)
R/S	13.8	15.1
H	0.398096597	0.411751313
D	1.601903403	1.588248687

Table 3.

Calculation of Fractal Dimension *D* of Annual Series of Standard Deviation of Corn Prices after the First Difference in the Logarithmic Values of Subsequent Prices

Years	*R/S*	$H = \lg(R/S) / \lg(365/2)$	$D = 2 - H$
1980	16.9	0.5430093	1.4569907
1981	16.2	0.5348847	1.4651153
1982	13.2	0.4955523	1.5044477
1983	17.5	0.5497097	1.4502903
1984	15.4	0.5251582	1.4748418
1985	10.4	0.4497634	1.5502366
1986	17	0.5441424	1.4558576
1987	21.8	0.5919066	1.4080934
1988	24.8	0.6166694	1.3833306
1989	18.1	0.5561842	1.4438158
1990	14.7	0.5162236	1.4837764
1991	16.4	0.5372413	1.4627587
1992	14.8	0.5175257	1.4824743

Table 3.

Years	R/S	H = lg(R/S) / lg(365/2)	D = 2 – H
1993	13.9	0.5054763	1.4945237
1994	17.1	0.5452688	1.4547312
1995	13.2	0.5532863	1.4467137
1996	22.5	0.6676436	1.3323564
1997	21.3	0.6558909	1.3441091
1998	19.6	0.6380548	1.3619452
1999	19.7	0.639146	1.360854
2000	19.9	0.6413121	1.3586879
2001	19.4	0.6358554	1.3641446
2002	19.6	0.6380548	1.3619452
2003	17.3	0.6112885	1.3887115
Value			
Average	17.52916667	0.5363751	1.4636249
For 23years (1988 – 2003)	17.5	0.5497097	1.4502903
Deviation	0.029166667	–0.0133345	0.0133345
Max	24.8	0.6166694	1.5502366
Min	10.4	0.4497634	1.3833306

Table 4.

Calculation of Fractal Dimension and Hurst Index for Annual Series of Price Volatility for the Corn Futures Contract in January 1980 – July 2003

	January (1988–2003)	February (1988–2003)	March (1988–2003)	April (1988–2003)	May (1988–2003)
R/S	15.3	12.7	14.4	15.3	18.5
H	0.400202106	0.372877327	0.391307894	0.400202106	0.42806488
D	1.599797894	1.627122673	1.608692106	1.599797894	1.57193512

	June (1988–2003)	July (1988–2003)	August (1988–2002)	September (1988–2002)	October (1988–2002)
R/S	21.7	26.3	22	17.9	17.2
H	0.451471148	0.479677042	0.453485499	0.423227862	0.431501618
D	1.548528852	1.520322958	1.546514501	1.576772138	1.568498382

	November (1988–2002)	December (1988–2002)
R/S	15.5	12.7
H	0.402107452	0.372877327
D	1.597892548	1.627122673

If the series grew during the previous period, it is likely to maintain this tendency for some time in the future, which is clearly illustrated by the above charts. Today's events will also be important tomorrow. This is the so called effect of long-term market memory.

However, the computations made on the basis of monthly charts for the same period (for example, January 1980–2003 and other months respectively) show that monthly price series at the corn market have different values of Hurst index and fractal dimension, ranging from 0.37 to 0.47 for Hurst index, and from 1.62 to 1.52 for fractal dimensions. Hence, this supports the E. Peters' conclusion that volatility should be anti-persistent.

The adopted values of Hurst indices and fractal dimensions correspond to persistent ergodic series. This type of a system is said to be «coming back to average». If the system demonstrates growth in the previous period, then there will most likely be a slump in the next one. As we see, yearly series are described as persistent trends, while monthly series are extraordinarily volatile with permanent reverses. The knowledge of these facts will help different participants of these markets, such as hedgers and speculators, to build their strategy.

The series of financial futures, such as the Dow Jones Industrial Average contract – the prediction indicator of the US stock exchange, have different values of Hurst index and fractal dimensions. The values of annual series until 1996 were $H = 0.54 - 0.41$, and after 1996 they made 0.57–0.62, which is rather close to random walk described by then-existing theories. Since today this market becomes persistent and has the long-term memory effect, the former theories can not explain its tendencies. Monthly series for the same period are anti-persistent, since the Hurst indices and fractal dimensions range from 0.37 (1.630) to 0.44 (1.56) respectively.

Therefore, our analysis shows how the computation of fractal dimensions allows to determine economic nature of the market and helps to determine its future tendencies with a high degree of probability. Fractal dimension provides an opportunity to learn market behaviour at different time horizons, which gives a fundamental scientific explanation of the efficiency of using current trading techniques (Elliot's wave theory, Fibonacci trading, Williams fractals, etc).

5. Applying Fractal Hypothesis to Ukrainian Futures Market

Proceeding from the previous research, we can assert that the hypothesis of futures market fractals as a tertiary segment of the financial market has all rights to exist, because of the following reasons:

- futures markets are chaotic and complex;

- futures markets are characterized by trends and long-term correlations (feed-back effect);
- critical levels of price changes are generated by certain factors at a certain period of time;
- time series of prices, profits, and their interrelations look similar or have fractal structure when time intervals decrease;
- reliability of predictions falls as future time intervals increase, that is, the financial markets are sensitive to initial conditions;
- in reality, futures markets are not in the equilibrium state.

In our opinion, the hypothesis of futures market fractals is far more reliable than the hypothesis of market efficiency. Therefore, it should be applied by participants in all segments of the financial market. In addition, this hypothesis should be used to change the concepts of financial and commodity markets development in Ukraine, as it evidently demonstrates the need for speculators and hedgers with different investment horizons (time fractals), since without them these markets will not become liquid.

This hypothesis needs to be actively implemented into practice by traders, who must master the calculation of fractal dimensions, which are more useful than standard deviations and mathematical expectation and will allow determining market risks and realize predictions of the market's future state.

With this end in view, the traders and those who wish to succeed at the futures market need to study thoroughly the innovative developments in fractal geometry, theory of chaos, neuron automatic design, etc. test them immediately on practice in order to create the proper empiric base for successive analysis and theoretical conclusions.

In the author's opinion, the non-linear and chaotic processes of national financial markets formation require new instruments, different from traditional methods of analysis, applied at the international markets. It is high time to talk not about national peculiarities, but about the **Non-linear Paradigm,** the most advanced innovative direction of research, which should be used to analyze national economic processes.

Therefore, today we need to consider which changes should be made in the financial managers' preparation programs. First of all, this refers to mathematical preparation of financiers. Mathematics should include fractal geometry and the chaos theory together with probability theory and econometrics, which today covers only the construction of linear models and one-sidedly describes real economy.

Similar ideas are also prevailing in natural sciences. Thus, R. May strongly recommends that people study non-linear processes at the early stage of their mathematics course. A good example of such a problem could be to solve for the population growth according to the Ferhulst equation (x_{t+1} =

$4a \cdot x_t \cdot (1 - x_t)$). This simple equation can be checked with a calculator. Testing a great number of simple equation solutions could substantially enrich the intuitive ideas about non-linear systems.

In our view, the financial theory should cover not only the linear models well-known to researchers (which are, by the way, studied superficially), but also everything that is new and only starting to manifest itself. At present time, such an innovation is the dynamic fractal model of the financial market, which allows to explain the market's slow and chaotic development.

Bibliography

1. Аналитика фондового рынка Украины // Бизнес. – № 29 (444) от 16 июля 2001 года. – С. 20.
2. Бестенс Д. Э., Ван дер Берг В. М, Вуд Д. Нейронные сети и финансовые рынки: принятие решений в торговых операциях. – М.: ТВП, 1997. – 236 .
3. Бункич М. К. Валютный рынок. – М.: «Ао 2 ДИС», 1995. – 112 с.
4. Вильямс Б. Новые измерения в биржевой торговле. – М.: ИК Аналитика, 2000. – 382 с.
5. Вильямс Б. Торговый хаос. – М.: ИК Аналитика, 2000. – 348 с.
6. Гитман Л. Дж., Джонк М. Д. Основы инвестирования / Пер. с англ. – М.: Дело, 1999. – 1008 с.
7. Долан Э. Дж., Линдсей Д. Рынок: микроэкономическая модель / Пер. с англ. – СПб., 1992. – 402 с.
8. Капица С. П., Курдюмов С. П., Малинецкий П. П. Синергетика и прогнозы будущевого. – www.keldysh.ru/depertments
9. Кидуэлл Д. С., Петерсон Р. Л., Блэкуэлл Д. У. Финансовые институты, рынки и деньги. – СПб.: Издательство «Питер», 2000. – 752 с.
10. Крушвиц Л. Финансирование и инвестиции. Неоклассические основы теории финансов / Пер с нем. – СПб.: Издательство «Питер», 2000. – 400 с.
11. Найман Э. Л. Малая Энциклопедия трейдера. – К.: Альфа-Капитал: Логос, 1997. – 236 с.
12. Найман Э. Л. Мастер-трейдинг: секретные материалы. – М.: Альпина Паблишер, 2002. – 320 с.
13. Найман Э. Л. Трейдер-Инвестор. – К.: ВИРА-Р, 2000. – 640 с.

14. Нили Г. Мастерство анализа волн Эллиота. – М.: Ик Аналитика, 2002. – 348 с.

15. Пайтаген Х., Рихтер П. Х. Красоты фракталов. – М., 1993. – С. 17–37, 39–42, 131–160.

16. Петерс Е. Хаос и порядок на рынках капитала. Новый аналитический взгляд на циклы, цены и изменчивость рынка: Пер. с англ. – М.: Мир, 2000. – 333 с.

17. Пректер Р., Фрост А. Дж. Волновой принцип Эллиота. Ключ к поведению рынка. – М.: Альпина Паблишер, 2001. – 268 с.

18. Пригожин И. От существующего к возникающему. Время и сложность в физических науках. – М., 1985. – С. 16–25.

19. Пригожин И., Стенгерс И. Время, хаос, квант. К решению парадокса времени. – М.: Прогресс, 1994. – С. 6–21.

20. Сергеєва Л. Н. Нелінійні моделі складних економічних систем – Автореферат дисертації на здобуття наукового ступеня доктора. – К.: ДНУ. – 32 с.

21. Сорос Дж. Кризис мирового капитализма. Отрытое общество в опасности: Пер. с англ. – М.: ИНФРА-М, 1999. – С. 237.

22. Сохацька О. М Концептуальні засади ціноутворення на ф'ючерсних ринках // Банківська справа. – 2000. – № 5. – С. 29–32, 63.

23. Сохацька О. М. Теоретичні основи функціонування ф'ючерсних ринків // Економіка України. – 2001. – № 12. – С. 55–61.

24. Тріллінберг В. Нобелівські премії в галузі економіки // Журнал європейської економіки. – 2002. – № 2. – С. 243–249.

25. Тьюлз Р., Брэдли Э., Тьюлз Т. Фондовый рынок. 6-е изд. / Пер. с англ. – М.: ИНФРА-М, 1997. – 648 с.

26. Федер Е. Фракталы. Пер. с англ. – М.: Мир, 1991. – 254 с.

27. Фишер Р. Трейдинг по Фибоначчи: практические приемы и методы. – М: ЕВРО, 2002. – 192 с.

28. Шарп У. Ф., Александр Г. Дж., Бейли Дж. В. Инвестиции: Пер. с англ. – М.: ИНФРА-М, 1999. – 1024 с.

29. Элдер А. Как играть и выигрывать на бирже. – М.: Диаграмма, 2001. – 352 с.

30. Якимкин В. Н Финансовый дилинг: Книга 1. – М.: ИКФ Омега. – Л., 2001. – 496 с.

31. Якимкин В. Н. Рынок Форекс – ваш путь к успеху. – М.: Светочь, 1999. – 256 с.

32. Fama E. Efficient Capital Market: A Review of Teory and EmpiricalvEvidence // Journal of Finance. – 1970.
33. Hicks J. R.Value and Capital. – London, 1939.
34. Keynes J. M. A Treatise on Money, vol 2, London, 1930.
35. Mandelbrot B. The Variation of Certain Speculative Prices in P Coother, ed., The Randon Character of Stock Prices. – Cambridje: MIT Press, 1964.
36. Markowitz H. M. Portfolio Selection Journal of Finance. – 1952. – № 1. – P. 77–91.
37. Peters E. E. Fractal Structure in The Capital Market // Financial Analists Journal. July-august, 1989.
38. www.cbt.com
39. www.ioa.rpd.univ.kiev.ua, www.fractal.cdu.edu.ua, www.keldysh.ru.

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