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Uwe HASSLER

TIME-VARYING VOLATILITY AND NON-STATIONARY TIME SERIES
– A Contribution to the Nobel Prize for Robert F. Engle and Clive W. J. Granger

This year's Nobel prize for Economics honours in a certain sense the University of California in San Diego (UCSD), where both Clive Granger and Robert Engle worked for a quarter of a century until last year. Clive Granger, born in Great Britain in 1934, studied mathematics and completed a Ph. D. in statistics in Nottingham. In 1974 he settled at the Department of Economics in San Diego. Robert Engle, born in 1942, graduated in physics and received a Ph. D. in economics at Cornell University. He joined San Diego in 1975. Today, Granger is Professor Emeritus of UCSD, and Engle has a chair at the Stern School of Business, New York University. Both laureates have revolutionized in the eighties of the last century the way economists analyze, model and predict time series, and gave rise to a change of paradigm in time series econometrics. The Nobel Committee honoured Engle for the introduction and development of models with time-varying conditional volatility (ARCH), while Granger was awarded the Nobel prize for creating a model framework allowing for dependencies between non-stationary variables (co-integration).

In the following, the major importance of these two concepts for business and economics will be elaborated. The focus will be on the path-breaking contributions of Robert Engle and Clive Granger. We shall not try to give a survey of both partly interrelated fields and, therefore, we do not pay tribute to important contributions of other authors following Engle and Granger. Since volatility

Hassler Uwe, Professor, Dr., Goethe University Frankfurt, Germany.

sometimes is modelled as non-stationary (integrated), we begin this contribution with Granger in order to refer to integration when discussing time-varying volatility.

Co-integration: Econometric Modelling of Non-stationary Time Series

Most economic data appearing in the course of time are non-stationary, more accurately display a trend. A very early paper stressing this is written by Granger himself[1]. Traditionally, non-stationarity was modelled at best by deterministic trends as means of time series growing more or less steadily. In the eighties of the last century, the idea gained ground that most of the economic time series follow a so-called stochastic trend, i.e. their variability increases over time. Such time series are called integrated. Co-integration then occurs when several integrated time series follow one and the same trend, or more generally, if there are several common trends in their background. Statistical inference, i.e. the basics of statistical estimation and testing, is radically different with integrated time series from the stationary case. Although non-stationarity of economic time series is the rule rather than the exception, no statistically appropriate econometric modelling existed up to the contributions of Granger. However, since the nineties of the last century no general introductory text book in econometrics was published without discussing co-integration.

Integrated Time Series

A time series is called integrated, when the differences, i. e. the increases in time, are stationary, or more accurately: If differencing is necessary to obtain stationarity. Here stationarity means the tendency of a time series to return to a fixed value. The level of an integrated time series in the current period is defined as the accumulation of stationary differences of past periods. An integrated time series does not have the tendency to oscillate around a certain level, but to drift above any value. Its variance increases over time, reflecting the difficulties of predicting: Basically, the best prediction of the value tomorrow is the value today, because there is no better imagination about the future course. The behaviour of many macro-economic series as well as financial time series such as e. g. consumption expenditure, exchange rates, interest rates or stock values

[1] C. W. J. Granger, The Typical Spectral Shape of an Economic Variable, in: Econometrica 34 (1966), 150–161.

can be approximated quite well that way, possibly after taking logarithms. However, this does not mean that the economy of a country is drifting arbitrarily apart. Obviously there are mechanisms tying together certain economic variables, so that they show a certain parallel movement (co-movement), i.e. they follow a common trend. This is the idea formalized by Granger under the notion of co-integration. Integrated time series for which there exists a stationary linear combination are called co-integrated. Economically speaking, there exists a long-run equilibrium relationship between non-stationary variables, and the deviations from the relationship are stationary, oscillating around zero.

If we take e. g. consumption expenditure of private households and national income as integrated, then there is a combination of both time series, namely savings as the difference between income and expenditure, which is stationary. Certainly, the last period savings influence consumption of the current period and – maybe – national income. In this sense, the non-stationary variables do not drift apart, but adjust to an equilibrium relationship because of former deviations from equilibrium, if a long-run, stable, economic relationship (co-integration) exists. These adjustment mechanisms are now famous in the literature under the notion error-correction models.

Error-correction and Causality

The representation theorem of Granger describes the following fact: The existence of co-integration of two or more variables is equivalent to the fact that the time series are generated by an error-correction model. Error-correction is the mirror image of co-integration. However, already in 1978 – before the concept of co-integration was born – an influential empirical study was published that related the differences of integrated consumption expenditure to stationary savings of the previous period in the framework of an error-correction model[2]. One reason why the concept of co-integration could succeed so quickly in the field of empirical economics was that a technical, statistical time series approach was combined with the concept of economic equilibrium.

Co-integration of several time-series implies, because of the dynamic error-correction, that the predictability of a variable increases through the knowledge of the past of the other variables. This concept of forecast improvement widely used by applied economists is called Granger causality[3]. It holds true that of two co-integrated time series at least one is Granger-causal to the other. Taking into account that one single integrated time series is very hard to be pre-

[2] J. E. H. Davidson, D. F. Hendry, F. Deba and S. Yeo, «Econometric Modelling of the Aggregate Time-Series relationship between Consumers Expenditure and Income in the United Kingdom», in: Economic Journal 88 (1978), 661–692.
[3] Cf. C. W. J. Granger, «Investigating Causal Relations by Econometric Models and Cross-Spectral methods», Econometrica 37 (1969), 424–438.

dicted, every forecast improvement is of practical relevance. A paper by Engle and Yoo[4] is devoted in particular to forecasts of co-integrated systems.

The concept of co-integration did not arise out of the blue, but found well prepared grounds. The notion and the mathematical formulation, however, was coined by Granger in 1981, popularized by Granger in 1986, and started its triumphal march with a paper by Engle and Granger 1987[5]. Today the vastly extended and elaborated co-integration methodology is a major workhorse in applied economics. The list of successful applications seems endless. The universal phenomenon of non-stationary time series justifies the exceptional status of the co-integration method.

Regression of Integrated Time Series

The statistical importance of co-integration must be appreciated with the problem of spurious regression in mind. This notion was coined by Granger and Newbold[6] in 1974 for the fact that statistically significant artificial relationships between independent integrated time series may show up due to the non-stationarity. In a simulation study, Granger and Newbold quantified the very high probability to detect incorrectly a relationship between independent integrated time series. The possibility of spurious regressions came as a shock to empirical economics. In an interview[7], Granger said that during a presentation of these results at the London School of Economics he met with disbelief and the suspicion of programming errors. After the publication of this paper, statistically careful economists estimated their models with stationary increments of time series, to avoid the danger of spurious regression. This led, however, to insignificant and economically implausible parameter estimates, because the equilibrium relationship of economic time series is between the levels of the variables. Therefore, time series econometrics was caught in a dilemma, either to risk spurious regressions between non-stationary variables or to produce insignificant results from the differences. The way out is co-integration: If and only if co-integration exists there is no danger of spurious regressions when regressing levels of integrated variables.

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[4] R. F. Engle and B. S. Yoo, «Forecasting and Testing in Co-integrated Systems», Journal of Econometrics 35 (1987), 143–159.
[5] C. W. J. Granger, «Some Properties of Time Series Data and their Use in Econometric Model Specification», Journal of Econometrics 16 (1981), 121–130; C. W. J. Granger, «Development in the study of Co-integrated Economic Variables», Oxford Bulletin of Economics and Statistics 48 (1986), Engle and Granger, «Co-Integration and Error Correction: Representation, Estimation, and Testing», Econometrica 55 (1987), 251–276.
[6] C. W. J. Granger and P. Newbold, «Spurious Regression in Econometrics», Journal of Econometrics 2 (1974), 111–120.
[7] «The ET-Interview: Professor Clive Granger», Econometric Theory 13 (1997), 253–303.

Hence, if there is co-integration then there is no spurious regression. This, however, does not mean that the standard results of stationary time series econometrics are valid. If there exists one and only one co-integration relationship between integrated variables, the least-squares regression of a single equation of levels shows especially nice results. Since the work of Engle and Granger in 1987 the fact that the parameter estimation tends more quickly to the true value than with stationary variables (super consistency) became commonly known. This result is noteworthy because it is true also when the residual term and the integrated regressor are correlated. Hence, co-integration may overcome the endogeneity (Haavelmo) bias: Despite possibly existing contemporaneous dependencies due to simultaneous relationships between several equations, a single equation model can be estimated consistently. This is in sharp contrast to results of stationary standard econometrics. However, these advantages came with a price, because without some modifications the estimators are not asymptotically normally distributed. This means, the usual t-statistics can not be used for significance testing. Nevertheless, Engle and Granger succeeded in their Econometrica paper to show a simple research strategy. In short, it is the following. Regress the levels of integrated time series and then test from the residuals the null hypothesis that a spurious regression was run. If this can be rejected, then using the residuals an error-correction model can be built, which shows the economic adjustment mechanism. In error-correction equations only stationary variables appear and, therefore, the standard inference of traditional textbooks is valid.

Extensions

It is obvious, that the co-integration methodology experienced many extensions. The most important of them is the multivariate generalisation. With more than two time series, the uniqueness of the co-integration relations can get lost, because two or more linearly independent relationships may exist. This disrupts the single-equation approach and requires multivariate modelling. Today it is standard to use to this end the vector-autoregressive (VAR) model.

Granger was engaged in several different extensions, three of which should be mentioned. In 1980 Granger and Joyeux[8] considered the possibility that economic time series are somewhere between classical stationarity and integration. This is called fractional integration. Here, time series could be stationary but show very strong persistence and extremely long lasting autocorrelation (models with long memory), so that they show quasi-local trends. Consequently, in the mentioned paper of 1981 Granger took into consideration also fractional co-integration. With seasonal time series, say in work with quarterly data, it can

[8] C. W. J. Granger and R. Joyeux, «An Introduction to Long-Memory Time Series and Fractional Differencing», Journal of Time Series Analysis I (1980), 15–29.

happen that taking ordinary differences does not result in stationarity but it might be necessary to take yearly differences to the quarter of the year before to get stationarity. These are called seasonally integrated time series, which can also be co-integrated[9].

Multi-cointegration[10] concept is also more economically motivated. Taking the above mentioned example of consumption and incomes, and supposing that the time series are integrated and that savings are stationary, then the present value wealth as cumulated savings of the past is also integrated by definition. And because of this, statistically there is the possibility of the second, not implausible co-integration relationship, namely between the three variables: consumption, income, and wealth.

There is nor sense to name spheres in which empirical co-integration analysis is successfully applied. A universal phenomenon of instationarity of economic time-series makes integration methodology of a special importance.

ARCH: Modelling of Time-varying Conditional Variances

Although the empirical application of Engle's original paper[11] of 1982 concerned a macroeconomic time series (inflation), his ARCH models are applied in particular to high-frequency financial time series. With financial market time series one observes that the volatility (or variance) fluctuates very strongly in the course of time: nervous market periods with extreme values are followed by more quiet periods characterized by moderate observations. Thus, there are typically clusters of volatility. This, however, is incompatible with the assumption of normally distributed data. Although such clusters of volatility in stock return time-series were recognised already in 1960s, the time series analysis and econometrics stuck to the model with constant variances over time. Engle broke with this tradition and can, therefore, be considered as a co-founder of a research field called today financial econometrics. Only few economists may share the following experience with Engle, i.e. to witness that their discovery or invention does not only change science within a short period of time, but also economic practice.

[9] Cf. S: Hylleberg, R. F. Engle, C. W. J. Granger and B. S. Yoo, «Seasonal Integration and Co-Integration», Journal of Econometrics 44 (1990), 215–228.

[10] C. W. J. Granger and T. W. Lee, «Multicointegration», in: Th. B. Foinby and C. F. Rhodes (eds), Co-Integration, Spurious Regressions, and Unit Roots: Advances in Econometrics, vol. 8, JALPress (1990), 71–84.

[11] R. F. Engle, «Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation», Econometrica 50 (1982), 987–1007.

ARCH

This acronym stands for Autoregressive Conditional Heteroscedasticity. Homoscedasticity indicatesin econometrics a classical assumption in regression models, namely that the variances of disturbances are equal at all points of time (or for every individual in case of cross-section analyses). Heteroscedasticity is given when the assumption of homoscedasticity is not fulfilled. However, Engle did not propose a model for unconditional but for conditional variance. Here, the conditional variance depends upon the own past of the time series, hence the term «autoregressive». Furthermore, the conditional variance or volatility of time series itself is a stochastic variable. In particular in Engle's ARCH model, the variance of the current period – given the data of the time series up to the previous period – is a function of the quadratic realizations of the past. This allows to model volatility clusters. If in recent periods there was only moderate movement on the market, i.e. there are only small (in absolute value) realizations, then the conditional variance is small today causing also in turn a moderate value. To the contrary, extremely positive or negative values in the previous period cause the conditional variance of the current period to be high, causing as a result extreme values today.

Talking about ARCH we mostly mean GARCH, i. e. Generalized ARCH models, introduced by Tim Bollerslev[12], a doctoral student of Engle. In GARCH models the conditional variance depends on the history of the process and additionally on the own history of volatility. This allows in general a more parsimonious parameterization than pure ARCH models. To underline the importance of (G)ARCH models let's remind the fact once more: In the jubilee volume (No. 100) of the Journal of Econometrics the paper by Bollerslev was celebrated as the second most cited paper of this journal up to the year 2001. The first most cited paper was the above mentioned paper by Engle and Yoo about forecasting co-integrated systems.

It may seem pedantic to talk always about conditional variance/volatility. But everything else would be wrong, because under certain parameter constellations the unconditional variance of an ARCH model is in fact stationary and, therefore, time independent. Moreover, this is not only of academic interest. Also the practitioner is interested in the conditional volatility, because his idea about e.g. the risk of an asset in the future is built upon the explicit recognition of the past quotation record.

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[12] T. Bollerslev, «Generalized Autoregressive Conditional Heteroscedasticity», Journal of Econometrics 31 (1986), 307–327.

Risk Measurement and Forecasts

The estimation of GARCH models is actually a theoretically founded formalization of things, financial analysts have done ever since: to determine the current risk of an asset based on the volatility given to the recent course of returns. Before the paper by Engle, people moved a window of fixed width through the past of the time series. A reasonable window width was 22 days with daily observations, because this is an average number of working days per month. Therefore, considering 22 consecutive days the variances of returns were calculated with rolling estimates. Such an estimation of the variance is based on 22 squared past values weighted with identical weights. Engle made this instrument to measure the conditional variance more flexible. With pure ARCH models the weights for the squared past values are estimated from the data and are not fixed. With GARCH models even the total past is used with weights tending geometrically to zero: The further back is an observation, the smaller is its influence on the present volatility.

The necessity to have a reliable risk measure is even higher today than 20 years ago. Firstly, the trade volume in options and similar financial derivates jumped up considerably. The price or value of an option depends on the volatility of the underlying asset. Secondly, the Basle Agreements (relating to supervision of banks) require that banks and other financial intermediaries hold a multiple of their «value at risk» (VaR) as equity. VaR here means the minimum loss to be expected for a certain time period. Such a VaR value is explicitly future oriented and makes volatility forecasts necessary, which today are mostly based on GARCH models. A very actual survey on the present level of researches made in the sphere of volatility forecasts can be found in the paper by Poon and Granger[13].

Serial Dependence through Volatility

Let's make as an example the returns of a share. According to the efficient market hypothesis the best predictor for tomorrows return is today's return, because the return of tomorrow is not correlated with the value of today. According to the efficient market hypothesis price changes cannot be forecasted. Nevertheless, the value of tomorrow must not be stochastically independent from today's value. If the return is generated by an ARCH or GARCH process, then the squared return of tomorrow is correlated with the squared value of today. Although there is no correlation with respect to the level of returns, the returns are connected via the square. This is so because the conditional volatility of tomor-

[13] S. H. Poon and C. W. Granger, «Forecasting Volatility in Financial Markets: A Review», Journal of Economic Literature 41 (2003). 478–539.

row is dependent on the squared return of today, and because the volatility of tomorrow is in turn responsible for the spread of return tomorrow.

This correlation of the squared or conditional volatility means for forecasts the following: Even if the best forecast of the value for tomorrow is the value of today, the forecast interval is varying, i.e. the interval in which the value of tomorrow will lay with a certain pre-specified probability. In other words: modelling ARCH effects will lead also in efficient financial markets to forecast improvements, even when this is reflected only in the width of forecast intervals. Taking into account serial dependence of ARCH time series via volatility it is not surprising, that ARCH models can be parameterized as autoregressive (AR) models of the squared data. And a GARCH model can be represented as an autoregressive moving-average (ARMA) model in the squares. This fact that the new (G)ARCH models can be represented quite familiarly as AR(MA) models in the squares was certainly one cause for the fast spread and theoretical acceptance of these models. Furthermore, it illustrates the way how to generalize GARCH models, namely in the same way as the usual ARMA models were generalized.

Above we talked about integrated time series, so it may come without surprise, that there are also integrated GARCH models[14], which represent the volatility as nonstationary processes. Also the idea of fractional integration (long memory) was transferred to the GARCH literature to model volatilities which may be stationary but show strong persistence[15].

GARCH effects can be recognized not only with observed time series but also with the residuals of regression models. If these residuals are not homoscedastic then usual interference about the slope parameter is invalid. Therefore, nowadays it is routine to test residuals also for ARCH in the misspecification analysis of regression models. A respective Lagrange multiplier test based on a simple auxiliary regression of the squared residuals was proposed already in the original paper by Engle. If ARCH effects are detected, every familiar software allows to do a maximum-likelihood estimation of the (G)ARCH parameters.

Applications and Extensions

Three applications of GARCH were mentioned: improved forecast intervals in efficient markets; correctly specified regression residuals and valid statistical inference in regression models; and risk measurement and risk forecasts, for example, for the determination of option prices. But not only option prices depend on the volatility of the underlying asset, but also the price of the asset itself can be seen as a function of its variance, because for riskier assets one ex-

[14] R. F. Engle and T. Bollerslev, «Modelling the Persistence of Conditional Variances», Econometric Reviews (1986), 1–50.
[15] Z. Ding, C. W. J. Granger and R. F. Engle, «A long Memory Property of Stock Market Returns and a New Model», Journal of Empirical Finance 1 (1993), 83–106.

pects on average higher returns. This was the motivation for the formulation of so-called ARCH-in-mean (ARCH-M) models[16]. However, not only the own volatility determines prices but also the covariance with a market portfolio. The desire to model such relationships led to increasingly complicated models, namely to multivariate modelling of GARCH effects[17].

It is no an exaggeration to say that further generalizations and variants of the (G)ARCH model have flooded the literature. Most of them show more or less interesting acronyms to which Engle added another, after twenty years of reviewing and summarising the literature[18]: YAARCH, Yet Another ARCH.

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[16] R. F. Engle, D.M. Lilien and R.P. Robins, «Estimating Time-Varying Premia in the Term Structure», Econometrica 55 (1987), 391–407.
[17] R. F. Engle and K. F. Kroner, «Multivariate Simultaneous Generalized ARCH», Econometric Theory 11 (1995), 122–150.
[18] R. F. Engle, «New Frontiers for ARCH models», Journal of Applied Econometrics 17 (2002), 425–446.